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Mathematical model for the evaluation of dielectric properties of inner layers of insulation in three-core belted cables

***Abstract**. This paper presents a discussion of properties of the modified inspection scheme, which is used in practice of evaluation of partial capacitances and dielectric dissipation factor of insulating layers in three-core power cables by using aggregate measurements. The applying of the proposed modification allowed to develop an alternative approach for the evaluation of partial capacitances and dielectric dissipation factor.*

Streszczenie.** W artykule omówiono właściwości zmodyfikowanego schematu kontrolnego, który jest wykorzystywany w praktyce do wyznaczania pojemności cząstkowych i współczynnika rozproszenia dielektrycznego warstw izolacyjnych w trójżyłowych kablach elektroenergetycznych za pomocą pomiarów agregatowych. Zastosowanie zaproponowanej modyfikacji pozwoliło na opracowanie alternatywnego podejścia do wyznaczania pojemności cząstkowych oraz współczynnika rozpraszania dielektrycznego. (**Model matematyczny do oceny właściwości dielektrycznych wewnętrznych warstw izolacji w trójżyłowych kablach z opaskami)

Keywords: partial capacitance, dielectric dissipation factor, diagnostics of power cables.
Słowa kluczowe: częściowa pojemność elektryczna, współczynnik rozpraszania dielektrycznego, diagnostyka kabli elektroenergetycznych.

Introduction

Electrical capacitance and dielectric dissipation factor (tgδ) are widely used in practice of diagnostics the quality of electrical insulation of the subjected to various destructive external factors power cables [1, 2]. The other wide spread area of applying of these parameters is regulation of technical characteristics of various cables during the production stage [3]. A detailed analysis of components of complex dielectric permittivity, determined by applying the corresponding values of electrical capacitance and tgδ, allowed to distinguish the dependent on level of moisture penetration into the insulation, types of dielectric material response [4]. The methods of dielectric spectroscopy are efficiently used not only for cables with cross-linked polyethylene insulation, but also for cables with paper-impregnated insulation. The established linkage between the minimum value of tgδ in the frequency domain and the content of moisture in paper insulation [5] allows to consider measurements of tgδ as a seminal approach for the diagnostics of power cables with paper-impregnated insulation. The equivalent scheme of a three-core power cable with paper-impregnated insulation contains several partial capacitances and, caused by the dielectric power losses, resistors. The evaluation of technical condition of such cables requires information about the individual values of these capacitances and tgδ.

For such cables the evaluation of individual dielectric properties of each insulating layer is complicated by the existence of parasitic capacitive coupling between the adjacent electrically conductive elements. Such capacitive coupling distorts the results of measurements of individual dielectric parameters of insulating layers, as these results appear to be affected by the properties of dielectric material, which separates the located nearby electrically conductive elements.

Direct measurements of dielectric parameters can be made by applying impedance meters with three electrodes, or by using the specialized capacitance-to-voltage converters, which allow to suppress the effect of parasitic partial capacitances [6-8]. However, the efficiency of such technical solutions may depend on values of parasitic capacitances and, consequently, may become inapplicable in some practical cases [9].

The applying of aggregate measurements also allows to determine the individual dielectric parameters of insulating layers and can be made by using impedance meters with two electrodes. Nevertheless, practical implementation of this approach also has certain disadvantages. One of such disadvantages is some complication of measurements. This complication takes place due to the necessity to measure several aggregate values of electrical capacitance and tgδ by using various inspection schemes. Moreover, in the case of random errors in measured aggregate values of electrical capacitance and tgδ, the evaluation of individual parameters of insulating layers by solving a system of a linear algebraic equations may lead to a substantial loss of accuracy. The applying of the least squares method for the minimization of the RMS error of solving the overdetermined system of linear algebraic equations allows to mitigate this mutual effect of measurement errors. However, such minimization of the RMS error also causes further complication of measurement procedures.

Mentioned difficulties of the described approaches urge the development of the other methods. One of such methods has been developed in the previous study [10] and implies the resistive grounding of an inspected layer of insulation with subsequent direct measurement of phase shift between the waveforms of voltage drop across the grounding resistor and across the inspected layer of insulation. The results of simulations have shown that in the case when the value of a grounding resistance is negligible, the value of phase shift between these waveforms approaches the value of phase shift between the flowing through the tested partial capacitance current and voltage drop across this capacitance. Such similarity allowed to determine the individual value of tgδ of tested partial capacitance. Nevertheless, the necessity of keeping the value of a grounding resistance at some level, which would allow to measure the voltage drop across this resistance, caused some bias of estimated values.

The objective of this paper is the removal of the described source of bias by using the properties of the modified inspection scheme.

Modified scheme for the inspection of properties of inner layers of insulation in three-core power cables

The development of an approach for the evaluation of partial capacitances and tgδ of insulating layers between the electrically conductive elements of a power cable will be made based on the presented in Fig.1 equivalent scheme of a three-core power cable with belted insulation. This equivalent scheme illustrates the existence of a capacitive coupling between the metal elements of a power cable.

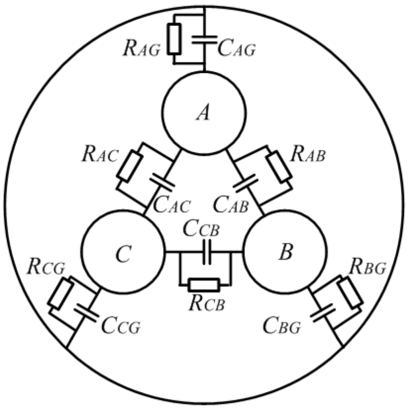

Fig.1. A conventional equivalent scheme of dielectric layers in a three-core power cable: C_{AB}, C_{CB}, C_{AC} – partial capacitances between the cores of a power cable, C_{AG}, C_{CG}, C_{BG} – partial capacitances between the sheath and cores *A*, *B* and *C* of a power cable, R_{AB}, R_{CB}, R_{AC} – resistors due to dielectric power losses in the layers of insulation between the cores of power cable, R_{AG}, R_{BG}, R_{CG} – resistors due to the dielectric power losses in the layers of insulation between the cores and the sheath of a power cable.

From the presented in the Fig. 1 equivalent scheme it can be concluded that the assessment of dielectric properties of inner layers of insulation requires the evaluation of 6 individual values of partial capacitances and 6 corresponding values of tgδ. For this purpose the presented approach will utilize the discussed in the previous study [10] inspection scheme, complemented with an auxiliary reference capacitor C_{k1}. This modified inspection scheme is presented in Fig. 2.

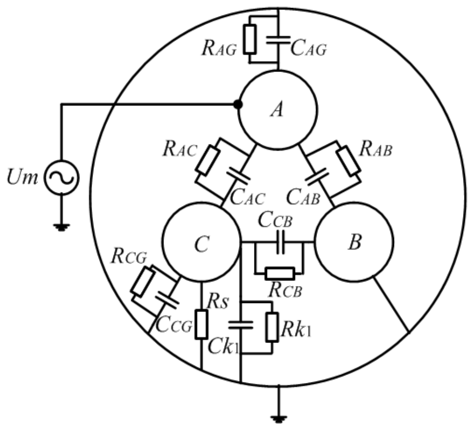

Fig.2. Modified inspection scheme of a three-core power cable: C_k – electrical capacitance of an auxiliary reference capacitor, R_k – shunt resistance, caused by dielectric power losses in the reference capacitor, R_s – auxiliary variable resistor.

The developed approach for measurements will be presented with respect to the layer of insulation between the cores *A* and *C* in the inspection scheme in Fig.2. According to this inspection scheme it can be concluded that the parts of the equivalent circuit, which contain the parameters of the inspected layer of insulation (R_{AC} and C_{AC}), are connected in parallel with respect to the brunches with parasitic parameters (R_{AG}, C_{AG}, R_{AB}, C_{AB}), whereas brunches with parasitic parameters R_{CG}, C_{CG}, R_{CB}, C_{CB} are connected in series with respect to the parameters of the inspected layer of insulation. In this case the values of voltage drop across the inspected layer of insulation between the cores *A* and *C* (U_{11}) and across the core *C* and the grounded sheath (U_{12}) can be correspondingly expressed according to the formulas [10]:

(1) $$U_{11} = \frac{UR_{AC}(j\omega C_{1k}R_{1k}R_s + R_s + R_{1k})}{R_{AC}R_s + R_{AC}R_{1k} + R_sR_{1k} + jA_1 + jA_2};$$

(2) $$U_{21} = \frac{UR_1R_s(j\omega C_{AC}R_{AC} + 1)}{R_{AC}R_s + R_{AC}R_1 + R_sR_1 + jA_1 + jA_2};$$

(3) $$\varphi_{U1} = arc\,tan\left(\frac{\sum_{i=1}^{3} g_i}{\sum_{i=4}^{10} g_i}\right);$$

(4) $$\varphi_{U2} = arc\,tan\left(-\frac{\sum_{i=11}^{13} g_i}{\sum_{i=14}^{18} g_i}\right);$$

where: ω – angular frequency of the applied voltage, *U* – r.m.s. value of the applied voltage, parameters g_1 ..g_{18} and A are expressed as follows:

(5) $$g_1 = \omega C_{1k}R_s^2R_{1k}^2$$

(6) $$g_2 = -\omega R_{1k}R_s^2C_{AC}R_{AC}$$

(7) $$g_3 = -\omega R_{1k}R_s^2C_{AC}R_{AC}$$

(8) $$g_4 = R_{AC}R_s^2$$

(9) $$g_5 = 2R_sR_{1k}R_{AC}$$

(10) $$g_6 = R_{1k}R_s^2$$

(11) $$g_7 = R_{AC}R_{1k}^2$$

(12) $$g_8 = R_{1k}^2R_s$$

(13) $$g_9 = \omega^2C_{1k}^2R_{1k}^2R_s^2R_{AC}$$

(14) $$g_{10} = \omega^2C_{1k}R_{1k}^2R_s^2R_{AC}C_{AC}$$

(15) $$g_{11} = \omega R_{AC}C_{1k}R_{1k}R_s$$

(16) $$g_{12} = -R_s\omega C_{AC}R_{AC}^2$$

(17) $$g_{13} = -\omega R_{AC}^2C_{AC}R_{1k}$$

(18) $$g_{14} = R_{AC}R_s$$

(19) $$g_{15} = R_{AC}R_{1k}$$

(20) $$g_{16} = R_{1k}R_s$$

(21) $$g_{17} = R_sR_{1k}\omega^2C_{AC}R_{AC}^2C_{1k}$$

(22) $$g_{18} = R_s\omega^2C_{AC}^2R_{AC}^2R_{1k}$$

(23) $$A_1 = \omega R_{AC}C_{1k}R_{1k}R_s$$

(24) $$A_2 = \omega R_sR_{1k}C_{AC}R_{AC}$$

The existence of the previously omitted auxiliary reference capacitor C_{k1}, as well as the presence of its shunt resistance R_{k1}, is taken into account by calculating the included in (1-4) parameters C_{1k} and R_{1k} as follows:

(25) $$C_{1k} = C_1 + C_{k1}$$

(26) $$R_{1k} = \frac{R_{CG}R_{CB}R_{k1}}{R_{CB}R_{k1} + R_{CG}R_{k1} + R_{CB}R_{CG}}$$

where the value of C_1 can be expressed as:

(27) $$C_1 = C_{CG} + C_{BC}$$

The developed approach for the evaluation of partial capacitance and tgδ of inner layers of insulation will be premised on the analysis of properties of the relations (1-4).

Properties of the modified inspection scheme and formation of system of equations for the evaluation of partial capacitances and tgδ

By assuming that $\varphi_{U1} = 0$ and solving the corresponding equality (3) with respect to R_s, it can be shown that the equality: $\varphi_{U1} = 0$ can be ensured either in the case of $R_s = 0$, or in the case when the value of R_s fits the stipulation:

$$R_{s1} = \frac{R_{1k} C_{AC} R_{AC}}{C_{1k} R_{1k} - C_{AC} R_{AC}} = \frac{C_{AC} R_{AC}}{C_{1k} - G_{1k} C_{AC} R_{AC}} \tag{28}$$

where G_{1k} can be expressed as:

$$G_{1k} = R_{1k}^{-1} = G_{k1} + G_1 = \frac{1}{R_{k1}} + \frac{R_{CB} + R_{CG}}{R_{CB} R_{CG}} \tag{29}$$

By substituting the expression (28) in (4), it can be established that the fulfillment of the condition (28) simultaneously ensures not only the equality: $\varphi_{U1} = 0$, but also the equality: $\varphi_{U2} = 0$. In addition, after the substitution of (28) in (1), it can be concluded that in the case of $\varphi_{U1} = \varphi_{U2} = 0$, the value of the voltage drop across the layer of insulation between the cores *A* and *C* of can be determined according to the expression:

$$U_{11RS} = \frac{U C_{1k}}{C_{1k} + C_{AC}} \tag{30}$$

where U_{11RS} is the value of voltage on the layer of insulation between cores *A* and *C*, determined in the case when the value of R_s satisfies the stipulation (22). By substituting (28) in (2), it can be established that for the case $\varphi_{U1} = \varphi_{U2} = 0$, the value of U_{21} can be determined as:

$$U_{21RS} = \frac{U C_{AC}}{C_{1k} + C_{AC}} \tag{31}$$

where: U_{21RS} – value of voltage U_{21} in the case when the value of R_s satisfies the stipulation (28). According to the expressions (30, 31), it can be concluded that in the case if R_s satisfies the stipulation (28), the value of voltage drop across the inspected layer of insulation between the cores *A* and *C* does not depend on the determined by the dielectric power losses elements (R_{AC}, R_1) of the inspection scheme in Fig. 2. Satisfaction of this stipulation also allows to ensure the independence of U_{11RS} and U_{21RS} on the value of an auxiliary resistor R_s. Consequently, by calculating the ratio of expressions (30) and (31), it can be shown that the value of this ratio is determined only by the ratio of partial capacitances C_1 and C_{AC}:

$$\frac{U_{11RS}}{U_{21RS}} = \frac{C_{1k}}{C_{AC}} = \frac{C_1 + C_{k1}}{C_{AC}} \tag{32}$$

If the capacitor C_{k1} is substituted with some other capacitor with its parameters C_{k2}, R_{k2}, the previously determined stipulation (28) will be attained in the case of some other value of an auxiliary resistance R_s. Consequently, this stipulation will be reformulated as:

$$R_{s2} = \frac{R_{2k} C_{AC} R_{AC}}{C_{2k} R_{2k} - C_{AC} R_{AC}} = \frac{C_{AC} R_{AC}}{C_{2k} - G_{2k} C_{AC} R_{AC}} \tag{33}$$

where: R_{s2} – value of the auxiliary resistance R_s which allows to ensure the equality: $\varphi_{U1} = \varphi_{U2} = 0$ and determined in the case if the capacitor C_{k1} is substituted with some other reference capacitor with its parameters C_{k2}, R_{k2}. The values of C_{2k} and G_{2k} are expressed as:

$$C_{2k} = C_1 + C_{k2} \tag{34}$$

$$G_{2k} = R_{2k}^{-1} = G_{k2} + G_1 = \frac{1}{R_{k2}} + \frac{R_{CB} + R_{CG}}{R_{CB} R_{CG}} \tag{35}$$

Variation of capacitance from C_{k1} to C_{k2} also leads to variation of voltage drop across the inspected layer of insulation and across the resistor R_{s2}. These values of voltage can be expressed as:

$$U_{12RS} = \frac{U C_{2k}}{C_{2k} + C_{AC}} \tag{36}$$

$$U_{22RS} = \frac{U C_{AC}}{C_{2k} + C_{AC}} \tag{37}$$

where U_{12RS} – value of voltage drop across the cores *A* and *C* in Fig. 2, provided the value of R_s fits (33),U_{22RS} – value of voltage drop across the resistor R_s provided its resistance fits (33). After the division of (36) by (37) we obtain similar to (32) expression, however, formulated after the substitution of C_{k1} with some other capacitor with capacitance C_{k2}:

$$\frac{U_{12RS}}{U_{22RS}} = \frac{C_{2k}}{C_{AC}} = \frac{C_1 + C_{k2}}{C_{AC}} \tag{38}$$

The solution of (32) with respect to C_{AC} yields:

$$C_{AC} = \frac{U_{21RS}\left(C_{k1} + C_1\right)}{U_{11RS}} \tag{39}$$

By substituting the derived relation for C_{AC} in (38) we obtain an expression which allows to calculate the unknown capacitance C_1.

$$C_1 = \frac{U_{12RS} U_{21RS} C_{k1} - U_{11RS} U_{22RS} C_{k2}}{U_{11RS} U_{22RS} - U_{12RS} U_{21RS}} \tag{40}$$

Calculation of C_1 by using (40) allows to determine the unknown value of partial capacitance C_{AC} according to (39).

After the calculation of C_1 and C_{AC} we can determine the unknown values of R_{AC}, and $tg\delta_{AC}$. By solving (28) with respect to R_{AC} we obtain the following relation:

$$R_{AC} = \frac{C_{1k} R_{s1}}{C_{AC}\left(G_1 R_{s1} + G_{k1} R_{s1} + 1\right)} \tag{41}$$

By substituting (41) in (33) and solving the obtained expression with respect to G_1 we obtain the relation:

$$G_1 = -\frac{C_{1k} G_{k2} R_{s1} R_{s2} - C_{2k} G_{k1} R_{s1} R_{s2} + C_{1k} R_{s1} - p}{R_{s1} R_{s2}\left(C_{1k} - C_{2k}\right)} \tag{42}$$

where *p* can be expressed as follows:

$$p = -C_{2k} R_{s2} \tag{43}$$

After the calculation of G_1 according to (42) we can determine the value of R_{AC} by using (41). Calculation of R_{AC} and C_{AC} allows to determine the value of dielectric dissipation factor of the inspected layer of insulation between the cores *A* and *C*. This calculation can be made by using the conventional expression for the value of dielectric dissipation factor of the represented by a parallel equivalent scheme capacitor with power losses:

$$tg\delta_{AC} = \frac{1}{\omega C_{AC} R_{AC}} \tag{44}$$

Consequently, the developed approach for the evaluation of C_{AC} and $tg\delta_{AC}$ implies the applying of the following sequence of procedures:

- grounding of the inspected layer of insulation through some variable resistor R_s and switched in parallel capacitor C_{k1} according to the presented in the Fig. 2 scheme.
- adjustment of R_s till the moment when the value of phase shift between the waveforms of voltage drop

across the inspected layer of insulation (U_{11}) and grounding resistor R_s (U_{21}) will be equal to zero.
- measurement of voltages U_{11}, U_{21} and grounding resistance R_{s1}.
- replacement of an auxiliary reference capacitor with another capacitor, with the determined in advance parameters $C_{к2}$ and R_{k2}.
- adjustment of R_s till the moment when the value of phase shift between the waveforms of voltage drop across the inspected layer of insulation (U_{12}) and grounding resistor R_s (U_{22}) will be equal to zero.
- measurement of voltages U_{12} and U_{22} and grounding resistance R_{s2}.
- calculation of C_1 according to (40)
- calculation of C_{AC} according to (39)
- consecutive calculation of G_1, R_{AC} and tgδ$_{AC}$ by using the expressions (42, 41, 44).

Results of simulations

Validation of the discussed properties of the presented in Fig. 2 inspection scheme, which allow to evaluate dielectric properties of insulating layers between the cores of a three-core power cable, can be made by using any kind of a circuit design software. For the purposes of convenience this inspection scheme in expanded form is presented in the Fig.3.

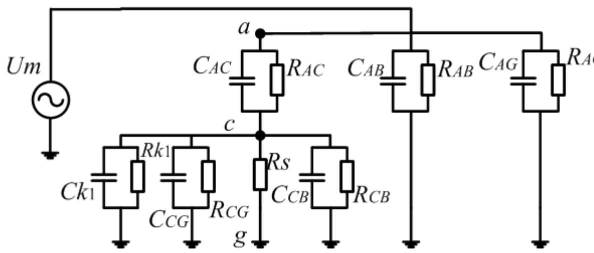

Fig.3. Expanded form of the modified inspection scheme presented in Fig.2

All simulations have been made for the presented in the Table 1 parameters of a power cable. The frequency of the applied voltage was equal to 1000 Hz and its RMS value was set to 10 V.

Table 1. The parameters of the inspection scheme in Fig. 3

C_{AG} [pF]	C_{BG} [pF]	C_{CG} [pF]	C_{AB} [pF]	C_{BC} [pF]	C_{AC} [pF]
1256	1321	1280	854	921	892
R_{AG} [MΩ]	**R_{BG} [MΩ]**	**R_{CG} [MΩ]**	**R_{AB} [MΩ]**	**R_{BC} [MΩ]**	**R_{AC} [MΩ]**
15.8394	14.8741	15.7392	22.4535	20.5722	21.7591
tgδ$_{AG}$	**tgδ$_{BG}$**	**tgδ$_{CG}$**	**tgδ$_{AB}$**	**tgδ$_{BC}$**	**tgδ$_{AC}$**
0.008	0.0081	0.0079	0.0083	0.0084	0.0082

Table 2. Calculated parameters and simulated values of voltage drop for the presented in Fig. 3 inspection scheme

Parameter	Value
R_{s1} [MΩ] (calculated by using (28))	1.0035
R_{s2} [Ω] (calculated by using (33))	457342
U_{11} (simulated) [V]	9.644
U_{21} (simulated) [mV]	355.46
U_{12} (simulated) [V]	9.822
U_{22} (simulated) [mV]	178
C_1 [pF] (calculated by using (27))	2182
C_{AC} [pF] (calculated by using (39))	891.3
R_{AC} [MΩ] (calculated by using (41))	21.776
tgδ$_{AC}$ (calculated by using (44))	0.00819

The parameters of auxiliary reference capacitors have been selected as follows: C_{k1} = 0.022·10^{-6} F, R_{k1} = 7.2343 MΩ, C_{k2} = 0.047·10^{-6} F, R_{k2} = 4.2328 MΩ.The results of calculations and simulations made in Multisim are presented in the Table 2.

The presented in the Table 2 results of calculations of parameters tgδ$_{AC}$ and C_{AC} exhibit a high level of proximity to the given in the Table 1 their exact values. Such proximity confirms that the unknown values of tgδ of insulating layers between the conductors of a three-core power cable can be evaluated by applying the developed approach.

Conclusions

The developed approach for the evaluation of tgδ and partial capacitance between the conductors of a three-core power cable is based on properties of the modified inspection scheme. The proposed modification implies the resistive grounding of an inspected layer of insulation and applying of an auxiliary reference capacitor, connected in parallel with respect to the grounding resistor. In further analysis special attention should paid to the affect of parameters of reference capacitor on the sensitivity of measurements.

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