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ANFIS Based Inverse Controller Design for Liquid Level Control of a Spherical Tank

***Abstract**. In this study, an adaptive neuro fuzzy inference system (ANFIS) based inverse controller design is presented for liquid level control application of a spherical tank. First, an excitation signal is applied to the system and the corresponding output signal is obtained. ANFIS-based fuzzy model of the nonlinear spherical tank system is constructed by using this input-output data set. While constructing the fuzzy model, a fuzzy model structure with two inputs and one output is preferred considering design simplicity. The input-output data used for constructing the fuzzy model of the system are exchanged, and by using this new data set, an ANFIS based inverse controller is designed. To improve the control performance against disturbances and model mismatches, the inverse controller is used in an internal model control structure. The performance of the proposed controller is compared to that of classical PI and fuzzy PI controllers under set point variation and disturbance conditions. The results of comparisons reveal that the proposed inverse controller outperforms both the classical and fuzzy PI controllers.*

***Streszczenie.** W niniejszym opracowaniu przedstawiono projekt regulatora odwrotnego opartego na adaptacyjnym neurorozmytym systemie wnioskowania (ANFIS) do zastosowania w kontroli poziomu cieczy w zbiorniku kulistym. Najpierw do systemu doprowadzany jest sygnał wzbudzenia i uzyskiwany jest odpowiedni sygnał wyjściowy. Oparty na ANFIS model rozmyty nieliniowego systemu zbiorników sferycznych jest tworzony przy użyciu tego zestawu danych wejściowych i wyjściowych. Podczas konstruowania modelu rozmytego preferowana jest struktura modelu rozmytego z dwoma danymi wejściowymi i jednym wynikiem, biorąc pod uwagę prostotę projektowania. Dane wejściowe-wyjściowe wykorzystywane do budowy modelu rozmytego systemu są wymieniane, a przy użyciu tego nowego zestawu danych projektowany jest sterownik odwrotny oparty na ANFIS. W celu poprawy wydajności sterowania w przypadku zakłóceń i niezgodności modelu, w wewnętrznej strukturze sterowania modelu zastosowano regulator odwrotny. Wydajność proponowanego regulatora jest porównywana z klasycznymi regulatorami PI i rozmytymi regulatorami PI w warunkach zmienności wartości zadanej i zakłóceń. Wyniki porównań pokazują, że proponowany regulator odwrotny przewyższa zarówno klasyczne, jak i rozmyte regulatory PI. (**Projekt sterownika ANFIS do sterowania poziomem cieczy w cylindtycznym zbiorniku**)*

Keywords: ANFIS, fuzzy logic, inverse controller, level control, spherical tank.
Słowa kluczowe: ANFIS, logika rozmyta, kontroler odwrotny, kontrola poziomu, zbiornik sferyczny.

Introduction

Proportional-integral-derivative (PID) controllers are widely used in industry since they have simple and effective structures. However, PID controllers are not able to provide effective control performance for nonlinear systems in general due to their linear characteristics. Model-based approaches have potential to provide more effective control performance than conventional PID controllers. However, obtaining an analytical model is not an easy task especially for highly nonlinear systems. Additionally, the analytical model can be unsuitable to be handled in controller design due to its complexity, even if it is obtained [1,2]. Fuzzy logic is an effective alternative for modeling and control of nonlinear systems since fuzzy systems are universal approximators [3-4]. For example, in nonlinear control applications, fuzzy PID controllers exhibit better performance than conventional PID controllers thanks to their nonlinear characteristics. Tuning parameters of PID and fuzzy PID controllers, such as gains, scaling factors, membership functions (MFs), etc, are usually set by trial and error method to obtain desired control performances. In complex and nonlinear systems, this trial and error approach can be tedious and time-consuming. In industry, controller tuning by trial and error method is often unacceptable considering safety, environmental and economic conditions. For such cases, it is necessary to use model-based control approaches instead of trial and error methods [5]. Since any nonlinear system can be modeled as a fuzzy system by using data-driven approaches, fuzzy models can effectively be utilized in a model-based controller design procedure.

One of the effective ways to design a nonlinear controller is to use the inverse of the system model directly, if it is available. However, obtaining an inverse definition of a nonlinear analytical model is a challenging task. Since the fuzzy modeling approach provides an easy way to obtain a model of a nonlinear system, several inversion approaches are proposed for fuzzy models in literature. These inversion methods can be divided into four groups; inversion of monotonic fuzzy models [6], inversion of decomposable fuzzy models [7-10] and inversion of more freely designed fuzzy models [11,12], and iterative inversion of the general class of fuzzy models [13,14]. Depending on the fuzzy model to be used, an inverse controller can easily be obtained by using one of these inversion methods. However, the inverse controller alone is not able to control the nonlinear system adequately when the process parameters change or when it is exposed to external disturbances. In these cases, different control structures are used such as model predictive control structure [5], internal model control (IMC) structure [7], and some online adaptation structures [14].

An adaptive neuro-fuzzy inference system (ANFIS), proposed by Jang [15], has been widely used in various modeling and control applications [16-20]. ANFIS is a particularly effective method for modeling nonlinear and complex systems since it requires minimal input and output training data, and learns quickly and accurately. Thanks to these properties, ANFIS can be used for inverse controller design as well. In literature, ANFIS based approaches are used for inverse controller design and tested on different nonlinear systems such as a vehicle suspension system [21], a coupled tank system [22], a stirrer tank reactor [23], an electro-hydraulic actuator system [24], and a dc motor [25].

In this study, inverse controller design for the liquid level control of a nonlinear spherical tank system is demonstrated. Forward fuzzy model is constructed based on ANFIS by using the collected input-output data set of the system. Then, exchanging the input and output data, the inverse controller is directly obtained again by using ANFIS. The forward fuzzy model and the inverse controller are used in an IMC structure to improve the robustness of the control system against disturbances and model mismatches. The effectiveness of the designed inverse controller is demonstrated through comparison simulations including classical PI and fuzzy PI controllers.

The rest of this paper is organized as follows: In Section II, the dynamic model of a spherical tank system is given. In Section III, the fuzzy model construction is presented. In Section IV, the inverse controller designed with ANFIS is illustrated. In Section V, simulation studies are demonstrated to show the superior performance of the designed inverse controller. Finally, in Section VI, the conclusions are given.

Dynamic model of a spherical tank system

Liquid level in a spherical tank system exhibits nonlinear characteristics. The general structure of a spherical tank system is demonstrated in Fig. 1. The governing dynamic equation for liquid level inside a spherical tank can be given as [26]

(1) $$Q_i\left(t-d\right)-Q_0=\left[\pi-\pi\left(R-y\right)^2\right]\frac{dy}{dt}$$

Here R, Q_i, and Q_o denote the tank radius, the inlet volumetric flow rate, and the outlet volumetric flow rate, respectively. y is the liquid level in the tank and d is the delay from Q_i to y. Considering the Bernoulli equation, the outlet volumetric flow rate can be defined as

(2) $$Q_0=\sqrt{2g\left(y-y_0\right)}$$

where y_o is the height of the output pipe centerline from the base of the spherical tank and g denotes the gravitational constant.

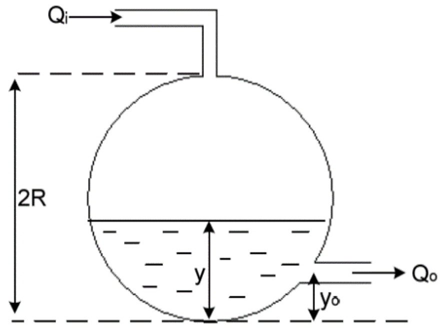

Fig.1. Spherical tank system

The system parameters of the tank used in this study are listed in Table 1. The inlet flow rate ranges from 0.1 to 6 m^3/s.

Table 1. Parameters of the spherical tank system

Parameters	Symbol	Value
Radius	R	1 m
Delay from Q_i to y	d	0
Gravitation constant	g	9.81 m/s^2
Height of the output pipe	y_0	0.1 m

Fuzzy Modeling of the Spherical Tank

The forward fuzzy model of the system is constructed to be used in an internal model control structure. In order to construct the fuzzy model of the tank system, the excitation signal given in (3) is applied to the system input and the corresponding output is obtained. The data is collected for 1000 s, and the sampling time is chosen as 0.1 s. The applied input and obtained corresponding output are demonstrated in Fig. 2a and Fig. 2b, respectively.

(3) $$Q_i=1.85\sin\left(0.02t\right)+0.3\sin\left(0.05t\right)+1.15\sin\left(0.08t\right)$$

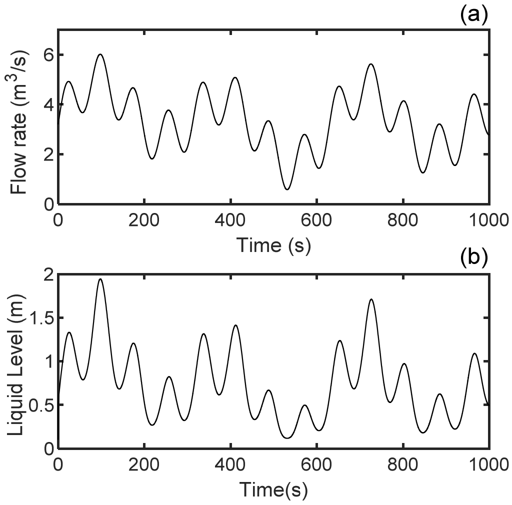

Fig.2. (a) Input signal (b) system output signal

Then, by using this input-output data set, ANFIS based forward model of the system is obtained. In order to provide design simplicity and also reduce the computational burden, a two-input-one-output Takagi-Sugeno fuzzy model structure is preferred as shown in Fig. 3. Three triangular MFs are used to define the antecedent fuzzy sets, and nine singleton MFs are used for the consequent fuzzy sets. The first 75% of the data are used for the training process and the remaining data are used for the validation process. The MFs and the rule table of the trained fuzzy model are illustrated in Fig. 4 and Table 2, respectively.

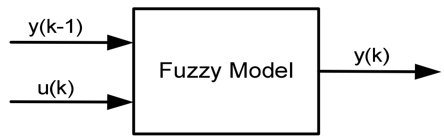

Fig.3. Input-output configuration of the fuzzy model

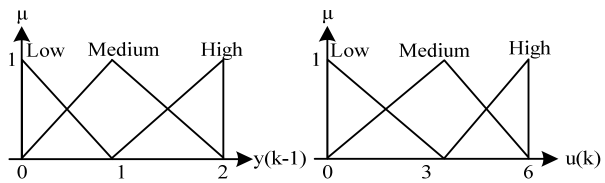

Fig.4. Antecedent MFs of the trained fuzzy model

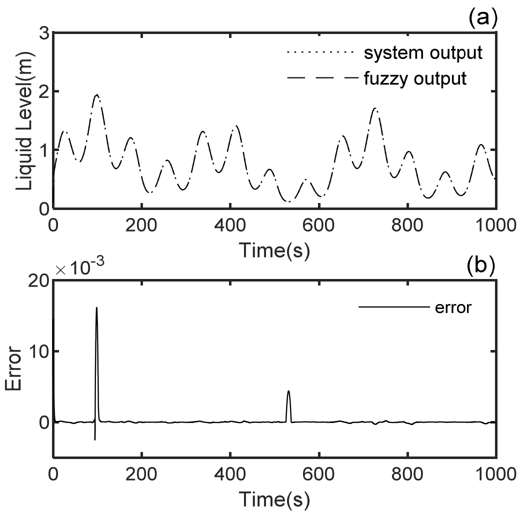

Fig.5. (a) Comparison of the system output and the fuzzy model output (b) modeling error of the constructed fuzzy model

Table 2. Rule table of the fuzzy model

y(k-1) / u(k)	Low	Medium	High
Low	0.024	0.085	0.173
Medium	0.760	0.924	1.021
High	0	1.808	1.912

The comparison of the outputs of the tank system and the constructed fuzzy model and also the obtained modeling error are given in Fig. 5a and Fig. 5b, respectively. The root mean square error (RMSE) value of the constructed fuzzy model is 2.1×10^{-3}, which indicates that acceptable modeling accuracy is obtained.

ANFIS Based Inverse Controller Design

In order to design the inverse controller, which also indicates the inverse model of the system, input and output data are exchanged and a new data set is defined. In this data set, *y(k)* and *y(k-1)* are used as inputs and *u(k)* is used as the output as shown in Fig. 6. The inverse controller is constructed by using ANFIS based on this new data set. Three triangular MFs and nine singleton MFs are used for the definitions of the antecedent fuzzy sets and the consequent fuzzy sets, respectively. As in the modeling procedure, the first 75% of the data and the remaining data are used for training and validation processes. The MFs and the rule table of the trained inverse controller are illustrated in Fig. 7 and Table 3, respectively.

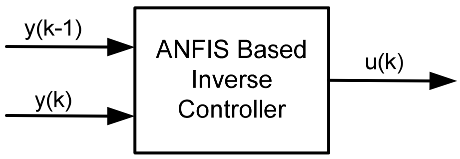

Fig.6. Input-output configuration of the ANFIS based inverse controller

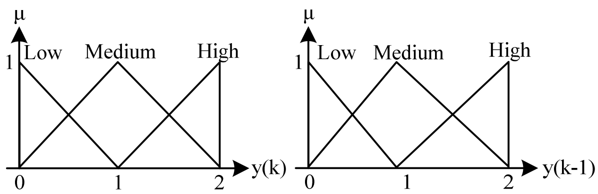

Fig.7. Antecedent MFs of the trained inverse controller

Table 3. Rule table of the inverse controller

y(k-1) / u(k)	Low	Medium	High
Low	2.394	-11.10	252.4
Medium	20.60	5.015	-17.45
High	0	29.120	5.962

The validation of the trained inverse controller is illustrated in Fig. 8. Fig. 8a shows the comparison of the applied input signal of the system and the derived input signal by the inverse controller for the same output values. The error signal is given in Fig. 8b. RMSE value of the constructed inverse controller is 5.8×10^{-2}. This indicates that acceptable accuracy is obtained.

In order to provide more robustness against disturbances and model mismatches, the designed inverse controller is used in the internal model control structure as shown in Fig. 9. In this structure, the difference between the system and the fuzzy model outputs is fed back to the controller. In this way, the controller can suppress the input and output disturbances and model mismatches.

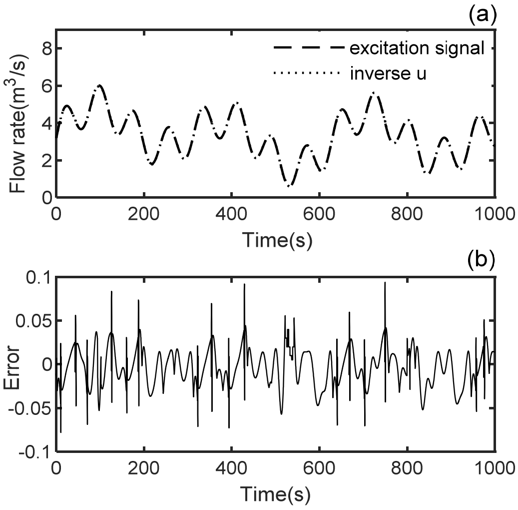

Fig.8. (a) Comparison of the system input and the inverse controller output (b) error of the constructed inverse controller

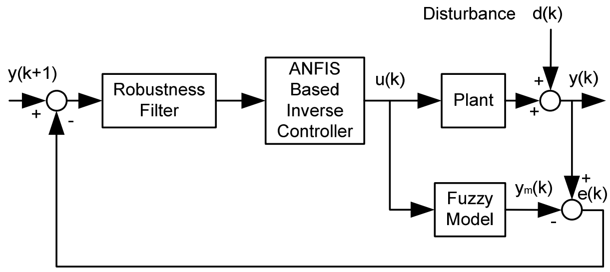

Fig.9. Proposed internal model control structure

Simulation Studies

A comparison study is performed under set point variation and disturbance conditions in order to evaluate the performance of the designed inverse controller. In the comparison, classical PI and fuzzy PI controllers are used. The parameters of both controllers are chosen so that the system response has an overshoot of 5% for the unit step reference signal. The parameters of the classical PI controller are determined as $K_p=1.4$ and $K_i=3.3$. The control output definition of the PI controller is given as

$$u(k)=\left(K_p+K_i\cdot T_s\frac{1}{z-1}\right)e(k) \quad (4)$$

Here *e(k)* is the error between the reference signal and the system response. T_s is the sampling time.

The Takagi-Sugeno fuzzy control structure with 3x3 rule base is chosen for the fuzzy PI controller. The structure of the fuzzy PI controller is given in Fig. 10.

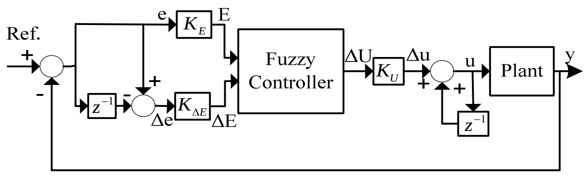

Fig.10. Structure of the fuzzy PI controller

Antecedent and consequent MFs and the rule table of the fuzzy PI controller are demonstrated in Fig. 11 and Table 4, respectively. The parameters of the fuzzy PI controller are determined as $K_E=0.65$, $K_{\Delta E}=0.25$ and $K_U=1$.

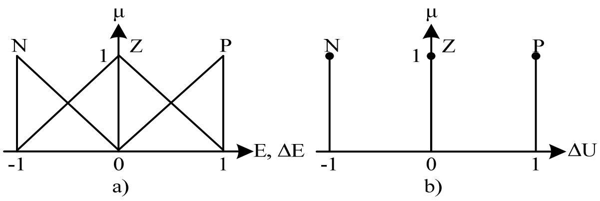

Fig.11. a) Antecedent and b) consequent membership functions

Table 4. Rule table of the fuzzy pi controller

E / ΔE	N	Z	P
N	N	N	Z
Z	N	Z	P
P	Z	P	P

To provide suitable control performance, the robustness filter for the inverse controller is chosen as follows

$$G_f(z) = \frac{0.1813}{z - 0.8187} \tag{5}$$

To evaluate the performances of the controllers, the most widely used performance indices integral square error (ISE), integral time square error (ITSE), integral absolute error (IAE) and integral time absolute error (ITAE) are used.

The control signals obtained by using three controllers and the related system responses are demonstrated in Fig. 12 for the unit step reference signal. The performance comparison of the controllers is given in Table 5.

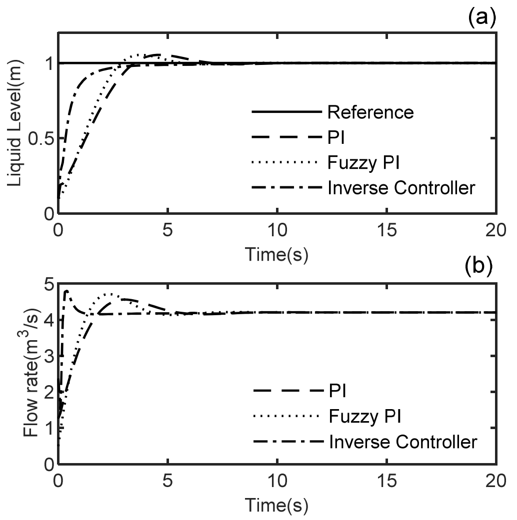

Fig.12. a) Comparison of the system responses for unit reference signal b) control signals

Table 5. Performance comparison for the unit step reference signal

	IAE	ITAE	ISE	ITSE
PI Controller	1.454	2.106	0.750	0.587
Fuzzy PI Controller	1.281	1.477	0.720	0.463
Inverse Controller	0.590	0.787	0.220	0.067

As it is seen from Fig. 12 and Table 5, the inverse controller outperforms the classical PI and fuzzy PI controllers for the unit step reference signal by providing the lowest IAE, ITAE, ISE, and ITSE values. The fuzzy PI controller shows better control performance compared to the classical PI controller.

The controller performances are evaluated under a set point variation condition. The applied reference signal, the corresponding outputs, and control signals are illustrated in Fig. 13. The performance comparisons are given in Table 6.

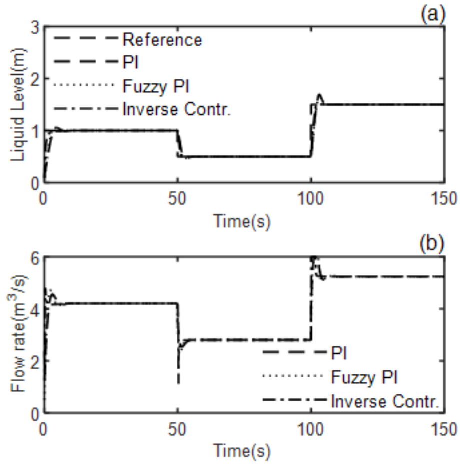

Fig.13. a) Comparison of the system responses under the set point variation condition b) control signals

Table 6. Performance comparison under the set point variation

	IAE	ITAE	ISE	ITSE
PI Controller	3.329	165.20	1.610	83.42
Fuzzy PI Controller	3.076	160.80	1.586	76.79
Inverse Controller	1.705	96.24	0.778	50.91

As it is seen from Fig. 13 and Table 6, the inverse controller exhibits significantly better performance compared to other controllers under the set point variation condition. The proposed controller also maintains its performance for different set points, while the other controllers show variable performances depending on the varying system gain at different levels of the spherical tank. Under the set point variation condition, the classical PI controller exhibits the lowest control performance.

Output disturbances are taken into consideration in order to evaluate the robustness performance of the proposed controller. The reference signal, the corresponding system responses, and applied output disturbances are demonstrated in Fig. 14a. The control signals and the comparison of the controller performances are given in Fig. 14b and Table 7, respectively. The performance values given in Table 7 denote the relative performance values with respect to the values in Table 6.

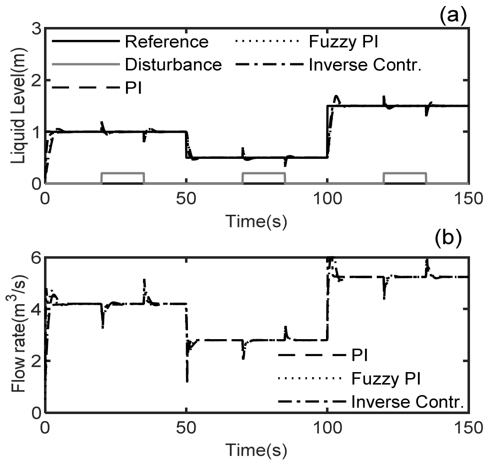

Fig.14. a) Comparison of the system responses under the output disturbance condition b) control signals

Table 7. Performance comparison under the output disturbance condition

	IAE	ITAE	ISE	ITSE
PI Controller	1.166	89.10	0.087	6.56
Fuzzy PI Controller	0.897	67.50	0.068	5.08
Inverse Controller	0.633	44.96	0.047	3.34

Under the output disturbance condition, the proposed inverse controller still exhibits better control performance compared to the other controllers as it is seen from Fig. 14 and Table 7. The fuzzy PI controller outperforms the classical PI controller under this condition as well.

Conclusion

In this study, an inverse controller design for the liquid level control of a nonlinear spherical tank system is demonstrated. The model of the nonlinear system under consideration is obtained in the form of a two-input-one-output Takagi-Sugeno fuzzy model. This fuzzy model is successfully trained with an acceptable error using generated input-output data set. The inputs and outputs in the data set are exchanged and a new data set is obtained to construct the inverse controller by using ANFIS. The trained inverse and forward models are used in an internal model control structure to provide robustness against disturbances and model mismatches. Simulation studies are performed and the results show that the performance of the designed inverse controller is better than that of classical PI and fuzzy PI controllers.

The design procedure of the proposed inverse controller is easy and straightforward. The forward and inverse fuzzy models of the system are constructed automatically by using ANFIS. Therefore, in this approach, there is no tuning parameter other than the single parameter of the robustness filter that can easily be tuned. Thus, no special tuning procedure is needed in the design procedure. Additionally, the given inverse control structure can easily be implemented for real-time applications since it includes two simple TS fuzzy systems with only two inputs. Considering these advantages of the inverse controller, this approach provides an effective alternative for control applications of nonlinear systems.

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