

## Modelling of an electric drive based on a DC motor and variable load influence examination

**Abstract.** A mathematical model of an electric drive created on the basis of a direct current motor with different types of excitation - independent, parallel and serial - was formed. Their characteristics were studied in the Simulink environment, and how taking into account the variable load of the electric drive caused by the arm of the robot affects this model. Such load is the source of the nonlinear component, which is introduced into the equation of motion of the electric drive and which must be taken into account in the design of the robot arm control system.

**Streszczenie.** Opracowano model matematyczny napędu elektrycznego, stworzony na bazie silnika prądu stałego o różnych rodzajach wzbudzenia – niezależnym, równoległym i szeregowym. Zbadano ich charakterystyki w środowisku Simulink oraz określono, jaki wpływ ma to na model uwzględniający zmienne obciążenie napędu elektrycznego wytwarzane przez ramię robota. Obciążenie jest źródłem składowej nieliniowej, która jest wprowadzana do równania ruchu napędu elektrycznego i którą należy uwzględnić przy projektowaniu układu sterowania ramieniem robota. (Modelowanie napędu elektrycznego opartego na silniku prądu stałego i badanie wpływu zmiennego obciążenia)

**Keywords:** direct, current, motor, modeling, mathematical.

**Słowa kluczowe:** prąd stały, silnik, modelowanie, matematyczny.

### Introduction

To successfully build an effective controller that should manage the process in an object, it is necessary to gather as much information as possible about the object itself and the physical nature of the processes that take place in it. The obtained information will help to choose the structure of the system that best meets the requirements, to determine the optimal modes of operation of the object, to correctly calculate the power of power elements to avoid significant overloads in the system and not increase its price.

However, not all factors can be considered. The nature of some factors is not always known; others may be random or have pronounced complex nonlinearities.

Certain features, the main of which is its inertia, when the value of the input value changes, a transient process takes place in it, characterize any real controlled dynamic object. In addition, in some cases, the object is subject to various perturbations, which leads to a change in the values of the output value at a constant value of the input. The system must function properly considering these factors. Therefore, when creating a controller, it is necessary to consider the properties of the controlled object, forming a mathematical model that would adequately reflect the processes occurring in this object.

It is known that the operation of the control system of electric motors is primarily affected by changes in load. In cases where mechanical links have to make complex movements according to a previously unknown law, the influence of their inertia becomes especially noticeable and decisive, which can also change when the relative position of these links' changes. For example, when the joints are bent, the static moments that occur due to the action of gravitational forces change, in addition. Namely, there are dynamic loads that depend on moments of inertia that are different at different positions of the joints. The weight, position of the link and the geometric dimensions of the load also affect the operation of the system and, in addition, they are not always strictly defined. Other mechanical factors have a significant impact on the operation of the system (a backlash, which changes as the gears work, the density of the oil, etc.) At variable loading on the engine the power consumed by it also changes that can cause its overheating and failure of both the engine and the power electric circuits providing management of its work.

Thus, the creation of a mathematical model of the object becomes an important part of the process of creating a controller. However, it can never fully reproduce a real object, due not only to the imperfection of the model itself, but also to the inability to accurately determine its parameters. The creation of a mathematical model of an object is based on information about the physical processes that take place in this object. As is known, the processing of models of physical systems occupies about 80 percent of the number of all operations required for the analysis and synthesis of control systems. In many cases, the creation of a mathematical model based on known theoretical dependences is significantly difficult, and when the nature of physical processes in the object is insufficiently studied, it is not possible to build an analytical model in general [1 - 7].

To avoid all the above difficulties will help an approach in which the model of the object is created solely based on data obtained in the process of its experimental research. To obtain such data, test signals are applied to the inputs of the object (the range of change of test signals corresponds to the range of change of input signals of the object) and the corresponding signals at its output are measured. Thus, it is possible to obtain the frequency characteristics of the object by applying sinusoids of different frequencies to the input, and then to use the representation of the object in the form of frequency characteristics to construct a corresponding controller. However, this approach is best suited for linear objects.

### Problem solving

During the study, various DC motors are modeled, namely: with independent, parallel and series excitation.

#### DC motor with independent excitation

To obtain a mathematical model of the motor, we consider the magnetization curve linear, and then the equation for the excitation winding has the form as follows:

$$(1) \quad L_e \frac{di_e}{dt} + r_e i_e = U_e$$

where:  $U_e$  – the voltage applied to the excitation winding;  $i_e$  – the excitation winding current;  $L_e$  – the inductance of the excitation winding;  $r_e$  – the resistance of the excitation winding.

The equation for the circle of the armature is written in the following form:

$$(2) \quad L_a \frac{di_a}{dt} + r_a i_a + e_a = U_a$$

where:  $U_a$  – the voltage applied to the armature;  $i_a$  – the armature current;  $L_a$  – the inductance of the armature winding;  $r_a$  – the resistance of the armature circuit;  $e_a$  – the opposite electromotive force that occurs during the rotation of the armature:

$$(3) \quad e_a = k\omega \Phi_e$$

where  $k$  – the constructive constant;  $\omega$  – the rotor speed;  $\Phi_e$  – the magnetic flux of the excitation winding.

The expression for determining the magnetic flux of the excitation winding has the form as follows:

$$(4) \quad \Phi_e = k_e i_e$$

where  $k_e$  – the constructive constant of the excitation circuit. The torque that occurs on the motor shaft is calculated by the following formula:

$$(5) \quad M = k_M i_a \Phi_e$$

where  $k_M$  – the structural constant of the motor.

The patterns of rotation of the motor shaft in the dynamics are determined by the following equation:

$$(6) \quad J \frac{d\omega}{dt} = M - M_L$$

To further simplify the equations of motor dynamics, we consider in more detail how energy is transmitted in a stationary mode from a source of electrical energy to a load. Given that in the steady state derivatives in the equations of dynamics are zero, we can assume that the passage of current through the excitation winding only leads to its heating. Then multiplying the left and right parts of equation

(2) by  $i_a$  and considering that  $\frac{di_a}{dt} = 0$ , we obtain as follows:

$$(7) \quad r_a i_a^2 + e_a i_a = U_a i_a$$

where:  $U_a i_a$  – the power of the armature winding source;  $r_a i_a^2$  – the part of the source power, which is spent on heating the armature winding;  $e_a i_a$  – the power to overcome opposite electromotive force and is converted into mechanical power, while providing torque.

Therefore, you can write as follows:

$$(8) \quad P_M = e_a i_a$$

From equation (6) it follows that  $M = M_L$ , if there is taken  $\frac{d\omega}{dt} = 0$ . Here we assume that the mechanical friction in

the motor is included in the value of the load moment. Then multiplying the left and the right parts of the expression  $M = M_L$ , by  $\omega$ , we obtain the expression  $\omega M = \omega M_L$  and according the following expression:

$$(9) \quad P = \frac{dA}{dt} = M \frac{d\phi}{dt} = M\omega,$$

we will write down  $P_M = M \cdot \omega$  (here  $A$  – the energy). Substituting the last equation in equation (8), we obtain  $M \cdot \omega = e_a i_a$  and considering expressions (3) and (5), we will have as follows:

$$(10) \quad k_M \omega i_a \Phi_e = k \omega i_a \Phi_e$$

whence it follows that  $k_M = k$ .

After substituting expressions (3), (4), (5) in equations (2) and (6) we obtain as follows:

$$(11) \quad L_a \frac{di_a}{dt} + r_a i_a + k \omega k_e i_e = U_a$$

$$(12) \quad J \frac{d\omega}{dt} = k_M i_a k_e i_e$$

From equation (10), it follows that  $k k_e = k_M k_e$ , therefore  $k k_e$  and  $k_M k_e$ , given that they have the dimensions of the inductors and entering the notation  $L_{ea} = k k_e$  we rewrite equations (2) and (6) in the following form:

$$(13) \quad L_a \frac{di_a}{dt} + r_a i_a + L_{ea} i_e \omega = U_a$$

$$(14) \quad J \frac{d\omega}{dt} = L_{ea} i_e i_a - M_L$$

Then, considering equation (1), we obtain a system of equations as follows:

$$(15) \quad \begin{cases} L_e \frac{di_e}{dt} + r_e i_e = U_e \\ L_a \frac{di_a}{dt} + r_a i_a + L_{ea} i_e \omega = U_a \\ J \frac{d\omega}{dt} = L_{ea} i_e i_a - M_L \end{cases}$$

Going to the representation in the operator form and regrouping the terms, we obtain a system of equations that can be used to create a model of the engine as follows:

$$(16) \quad \begin{cases} (sT_e + 1)i_e = \frac{U_e}{r_e} \\ (sT_a + 1)i_a = \frac{U_a - L_{ea} i_e \omega}{r_a} \\ s\omega = (L_{ea} i_e i_a - M_L)/(J) \end{cases}$$

where  $T_e$  and  $T_a$  – the time constant of the excitation and armature circuit; respectively.

### DC motor with serial excitation

To model a DC motor with serial excitation, it should be considered that it is as follows

$$(17) \quad \begin{cases} u_c + u_a = u; \\ i_c = i_a = i \end{cases}$$

Where  $u$  and  $i$  is the is the control voltage and control current, respectively.

Then equations (15) take the form as follows

$$(18) \quad \left. \begin{cases} (L_a + L_e) \frac{di}{dt} + (r_a + r_e) i + L_{ea} i_e \omega = u \\ J \frac{d\omega}{dt} = L_{ea} i^2 - M_L \end{cases} \right\}$$

By moving to the representation in the operator form and rearranging the terms, it will get the equation (18) in a following form convenient for creating the model

$$(19) \quad \left. \begin{cases} (sT + 1)i = (u - L_{ea} i \omega)/(r_a + r_e) \\ s\omega = (L_{ea} i^2 - M_L)/J \end{cases} \right\}$$

where  $T = (L_a + L_e)/(r_a + r_e)$  is the time constant of the excitation circuit.

### Simulation Results

All kinds of DC motor were implemented in the Simulink environment [8]. The “Manual Switch” element (Fig. 1) provides the ability to simulate both independently excited and parallel excited motor modes.

Before starting the simulation, we set the values of the parameters:  $Lzj=1.8$ ,  $Rz=1$ ,  $Rj=1$ ,  $Tz=0.5$ ,  $Tj=0.02$ ,  $J=1$ . The results of the motor with independent excitation modeling are shown in Fig. 2. This figure shows the time diagrams of

the signals at the control inputs  $U_c$ ,  $U_a$  and at the input of the perturbation  $M_L$ , as well as diagrams of the output values, i.e., rotor speed  $\omega$  and motor torque  $M$ . Additionally, time diagrams of currents in the control circuits are given  $i_c$ ,  $i_a$ .

When simulating a DC motor with independent excitation, the armature voltage is applied with a delay (a

time 5 sec) so that the transient process in the excitation circuit has time to complete. At the point  $t = 10$  sec, the load moment is given, which makes it possible to estimate the transient process caused by the perturbation.

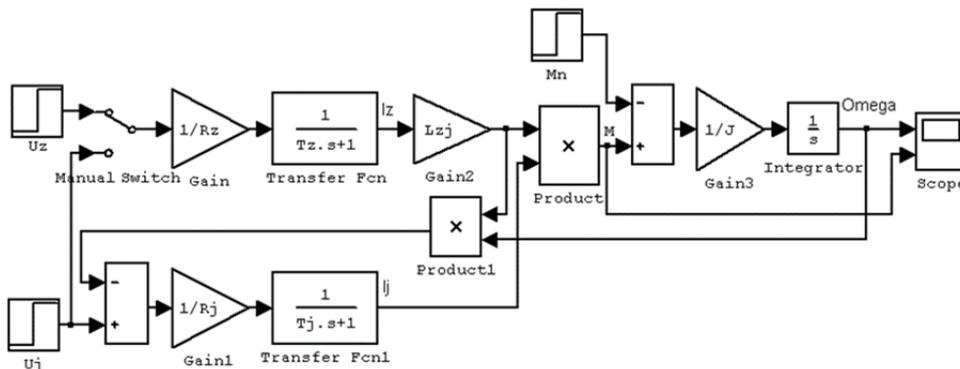


Fig. 1. The DC motor with independent excitation modelling

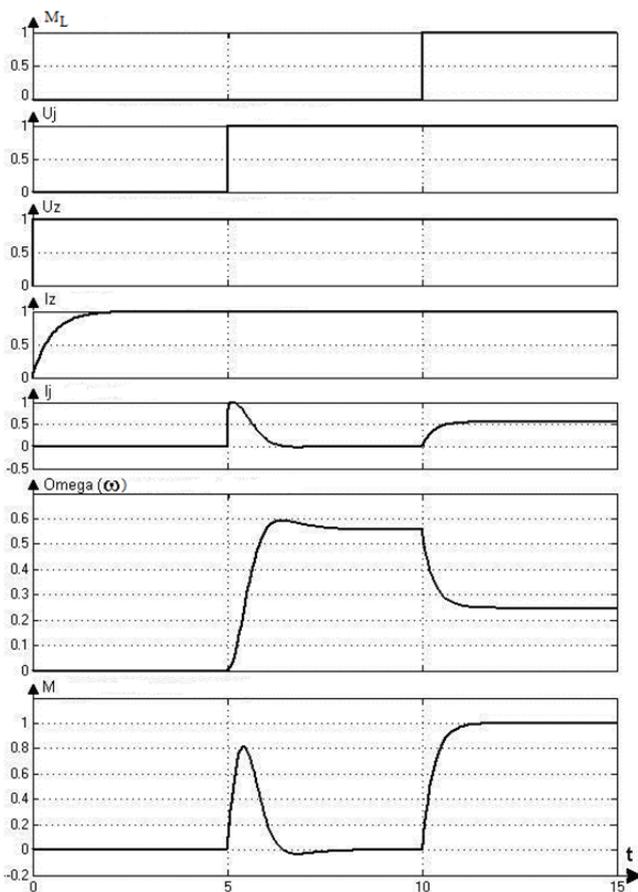


Fig. 2. The DC independent excited motor modelling results

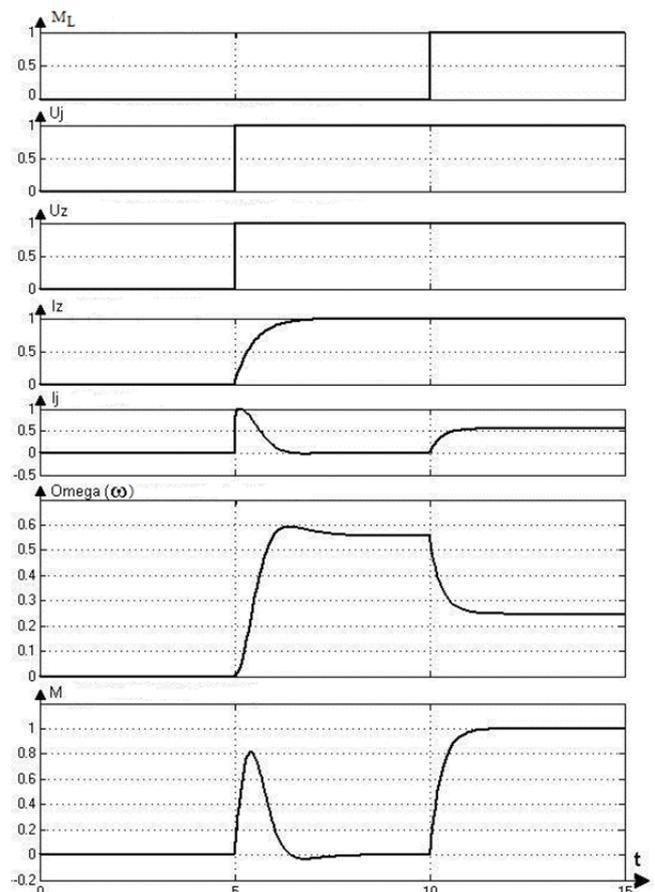


Fig. 3. Motor with parallel excitation simulation results

When modeling a motor with parallel excitation, voltage  $U_z$  is simultaneously applied to the armature and to the excitation circuit (to ensure  $u_a = u_c$  in the model (Fig. 1), for this purpose the **Manual Switch** element is used). The obtained simulation results are presented at Fig. 3.

The model of a DC motor with serial excitation, implemented in the Simulink system, is shown in Fig. 4. Before starting the simulation, it was set the parameter values:  $Lzj=1.8$ ,  $Rz=1$ ,  $Rj=1$ ,  $T=0.26$ ,  $J=1$ . The simulation results for the motor with serial excitation are shown in Fig. 5.

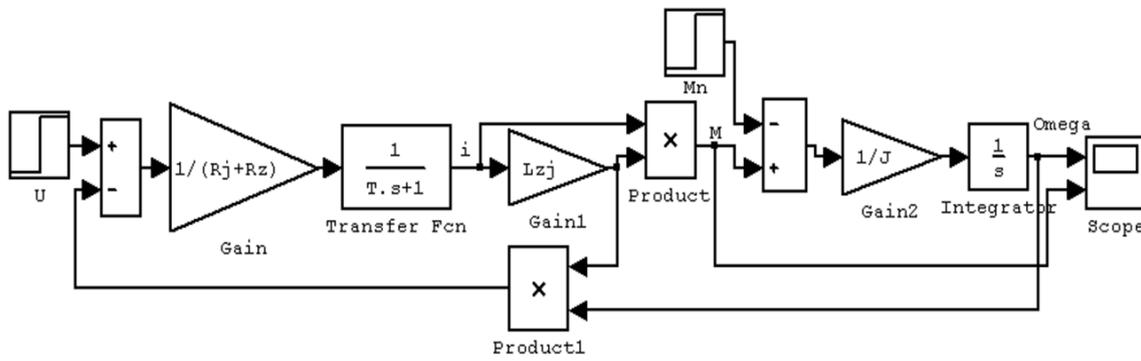


Fig.4. Model of a DC motor with serial excitation

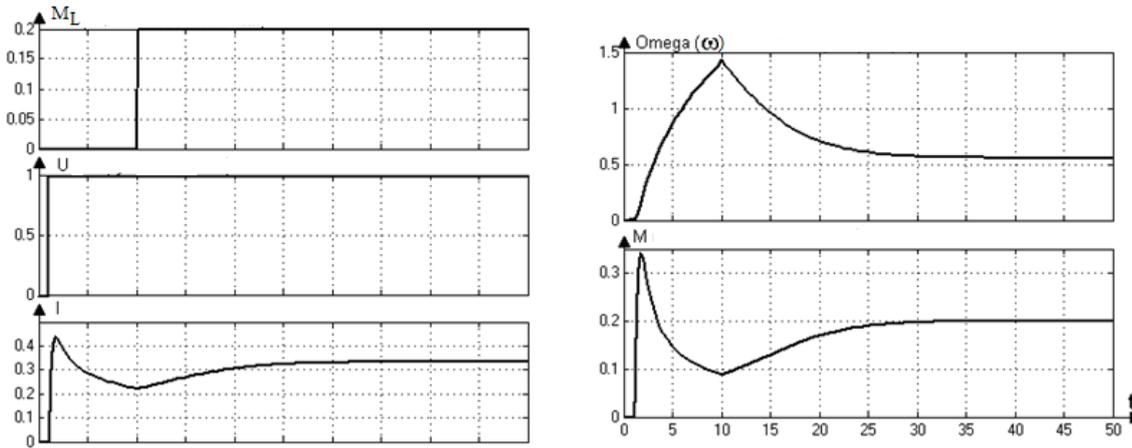


Fig.5. Simulation results of the DC motor with serial excitation modelling

The placement of the diagrams is aligned on top of each other, which allows us to see how the input signals are displayed on the output.

To estimate the mechanical characteristics of the above-described types of motors (independent, parallel and series excitation), it was fixed the voltage values at the control inputs, and slowly increased the load moment, adding to the diagrams in Fig. 1 and Fig. 4, the  $M_L$  element is supplied with a scaling element and an integrator, as shown in Fig. 6 as follows.

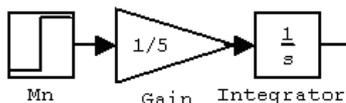


Fig. 6. The scheme of the formation of a linearly increasing load moment

The mechanical characteristics obtained during the study of direct current motor models with independent, parallel and serial excitation are shown in Fig. 7-9, respectively.

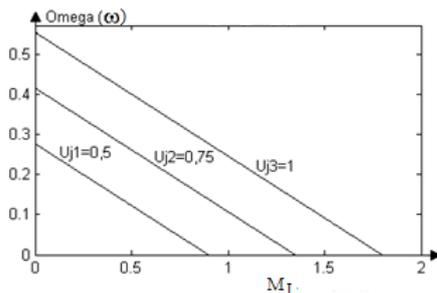


Fig. 7. Mechanical characteristics of a DC motor with independent excitation

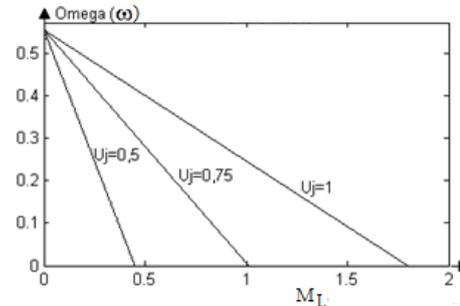


Fig. 8. Mechanical characteristics of a DC motor with parallel excitation

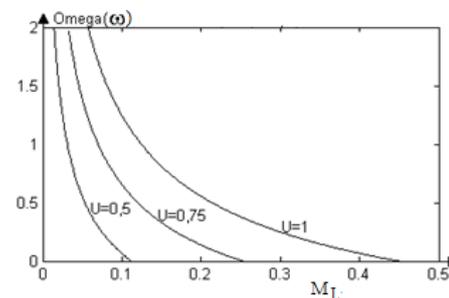


Fig. 9. Mechanical characteristics of a DC motor with serial excitation

From the results obtained in the modelling process, it follows that the mathematical model of the motor independent excitation adequately reproduces the processes that take place in a real object.

### Variable loading examination

As the next step let us consider a motor with independent excitation, which is loaded with a moment of viscous friction and a positional moment such as the rotation of the robot's arm [9 - 11]. In this case, the total moment can be given as the sum of the moment of the viscous friction  $M_{vf}$  and the moment of loading  $M_L$ , i.e.,

$$(20) M_{WM} = k_{vf1} \omega_{WM} + mga \sin \phi_{WM} = k_{vf1} \frac{\omega}{i} + mga \sin \frac{\phi}{i}$$

where  $\omega_{WM}$  – the speed of the working mechanism,  $\phi_{WM}$  – the angle of rotation of the working mechanism,  $i$  – the gear ratio of the gearbox,  $\omega$  – the motor shaft speed,  $\phi$  – the motor shaft angle of rotation,  $k_{vf1}$  – the coefficient of the viscous friction,  $m$  – the mass loading,  $g$  – the gravitation acceleration,  $l$  – the distance between the loading centre of mass and the arm axis.

If there are known the natural resistance of the motor  $M'$  the resistance of the working mechanism  $M_{WM}$ , the resistance of the gearbox  $M_r$  (it is supposed that it is concentrated on the leading side of the transmission) and the efficiency of the gearbox  $\eta_r$ , then for the total torque reduced to the motor shaft, obsessed as follows:

$$(21) M_L = M' + \frac{M_r + M_{WM}}{i \eta_r}$$

Then, according to (20) it can be written as follows:

$$(22) M_L = M' + \frac{M_r + k_{vf1} \frac{\omega}{i} + mga \sin \frac{\phi}{i}}{i \eta_r}$$

It is supposed that the motor natural moment of resistance, as well as the moment of resistance of the gearbox can be mainly represented by the moment of the motor viscous friction ( $M' = k'_{vf} \omega$ ), and the moment of the gearbox viscous friction ( $M_r = k_{vf2} \omega / i$ ). Having made substitution, and having grouped members in the right part of the expression (22) it is received as follows:

$$(23) M_L = k_1 \omega + k_2 \sin \frac{\phi}{i}$$

where  $k_1 = k'_{vf} + \frac{k_{vf2} + k_{vf1}}{i^2 \eta_r}$  and  $k_2 = \frac{mga}{i \eta_r}$  are some coefficients.

For the father simplification without changing the essence of equation (23), it is assumed that the gear ratio  $i = 1$  then this equation will be rewritten as following:

$$(24) M_L = k_1 \omega + k_2 \sin \phi$$

Substituting expression (24) into the system (15) it is obsessed as follows:

$$(25) \begin{cases} L_e \frac{di_e}{dt} + r_e i_e = U_e \\ L_a \frac{di_a}{dt} + r_a i_a + L_{ca} i_e \omega = U_a \\ J \frac{d\omega}{dt} = L_{ca} i_e i_a - k_1 \omega - k_2 \sin \phi \end{cases}$$

Since, for a DC motor with independent excitation, it can be assumed that  $\frac{di_e}{dt} = 0$  and  $i_e = \frac{U_e}{r_e}$ , then (25) is given as:

$$(26) \begin{cases} L_a \frac{di_a}{dt} + r_a i_a + L_{ca} \frac{U_e}{r_e} \omega = U_a \\ J \frac{d\omega}{dt} = L_{ca} \frac{U_e}{r_e} i_a - k_1 \omega - k_2 \sin \phi \end{cases}$$

Given that,  $\omega = \frac{d\phi}{dt}$  we obtain:

$$(27) \begin{cases} u_a = L_a \frac{di_a}{dt} + r_a i_a + L_{ca} \frac{U_e}{r_e} \frac{d\phi}{dt} \\ J \frac{d^2 \phi}{dt^2} = L_{ca} \frac{U_e}{r_e} i_a - k_1 \frac{d\phi}{dt} - k_2 \sin \phi \end{cases}$$

If in the first equation of the system (27)  $L_a$  is equated to zero (which is close to reality), then after carrying out certain transformations the second equation of this system can be written in the following form:

$$(28) \frac{d^2 \phi}{dt^2} + a \frac{d\phi}{dt} + b \sin \phi = c U_a$$

where

$$(29) a = \left( k_1 + \left( L_{ca} \frac{U_e}{r_e} \right)^2 / r_a \right) / J, \quad b = \frac{k_2}{J}, \quad c = L_{ca} \frac{U_e}{r_e r_e J}$$

Thus, the resulting equation of a DC motor with independent excitation, considering the moment of viscous friction and the moment of the type of rotation of the robot, is a nonlinear differential equation of the second order. Linearization of the obtained equation leads to its significant simplification, but it does not consider the nonlinearity of the motor characteristics, as one of the features that most significantly affects the behaviour of the motor, both in transient and stationary modes. If the creation of a mathematical model of the motor takes into account the influence of the inductance of the armature, the motor will be described by a higher order differential equation and with a much more complex nonlinear dependence, which significantly limits the use of such a model in solving problems related to analysis and synthesis of automatic control. So, the concrete modelling results depend on concrete loading nature (in our case it is the robot's arm) [12 - 15]. In this non-linearity equation case, adequate results are provided using the artificial neural network and fuzzy logic methodologies [16 - 19]. Also, there is known DC motor modeling using the Lagrange and Euler-Lagrange approach on contrary to the typical Kirchhoff's' and Newton laws [20].

### Conclusion

1. The dynamics of the DC motor with different excitation modes are analysed and it is shown that considering the action of the torque of the robot, the motor is described by a nonlinear differential equation of the second order.
2. Considering additional factors that significantly affect the operation of the motor leads to the complication of its mathematical model.
3. The rejection of certain dependences makes it possible to represent the motor model using a simplified system of differential equations, however, since the most significant dependencies cannot be ignored, these equations remain nonlinear.

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