1. Zahir Asradj [1,2], 2. Yassine Bensafia[2], 3. Imad Merzouk [1]

Applied Automation and Industrial Diagnostics Laboratory, Faculty of Science and Technology, University of Djelfa, 17000, Algeria (1),
Electrical Engineering Department, Faculty of Sciences and Applied sciences, University of Bouira, 10000 - Algeria (2),
ORCID: 1. 0000-0003-1760-3636; 2. 0000-0002-4062-5086; 3. 0000-0003-1539-1414

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Performances Improvement of Fractional order Systems using the Fractional Order Adaptive PID Controller

***Abstract**. Fractional order systems are widely used in industrial application for its different advantage such us high efficiency, and flexibilities. The applications of fractional order systems in a range of scientific fields have caught the attention of researchers especially in control strategy. The current research work presents the use the fractional adaptive PID controller approach, optimized by genetic algorithm, to improve the performances (rise time, setting time and overshoot) for fractional systems by introducing fractional order integrator and differentiator in the classical feedback adaptive PID controller. To validate the arguments, effectiveness and performances analysis of the proposed approach optimized by genetic algorithm have been studied in comparison to the classical adaptive PID controller. Numerical simulation and analysis are presented to verify the best controller. The Fractional order PID gives the best result in terms of settling time, rise time, overshoot and mean absolute error.*

***Streszczenie.** Systemy ułamkowego rzędu są szeroko stosowane w zastosowaniach przemysłowych ze względu na różne zalety, takie jak wysoka wydajność i elastyczność. Zastosowania systemów rzędu ułamkowego w wielu dziedzinach nauki przykuły uwagę badaczy, zwłaszcza w dziedzinie strategii sterowania. Obecna praca badawcza przedstawia wykorzystanie podejścia ułamkowego regulatora adaptacyjnego PID, zoptymalizowanego przez algorytm genetyczny, do poprawy osiągów (czas narastania, czas ustawiania i przeregulowanie) układów ułamkowych poprzez wprowadzenie integratora i układu różniczkowego ułamkowego rzędu do klasycznego regulatora PID z adaptacyjnym sprzężeniem zwrotnym. Aby zweryfikować argumenty, przeprowadzono analizę skuteczności i wydajności proponowanego podejścia zoptymalizowanego za pomocą algorytmu genetycznego w porównaniu z klasycznym adaptacyjnym regulatorem PID. Przedstawiono symulację i analizę numeryczną w celu weryfikacji najlepszego sterownika. PID rzędu ułamkowego daje najlepsze wyniki pod względem czasu ustalania, czasu narastania, przeregulowania i średniego błędu bezwzględnego. (**Wydajność Poprawa systemów ułamkowego rzędu przy użyciu adaptacyjnego regulatora PID ułamkowego rzędu**)*

Keywords: Fractional System, Fractional Adaptive PID controllers, Genetic Algorithm, Comparative performance analysis.
Słowa kluczowe: Systemy Układ ułamkowy, Ułamkowe adaptacyjne regulatory PID, Algorytm genetyczny, Analiza porównawcza wydajności.

Introduction

Fractional-order (FO) differential calculus, or fractional calculus, is an area of mathematics concerned with the expansion of well-known differentiation and integration operations to arbitrary orders. Thanks to correspondence between two mathematicians, Leibniz and L'Hospital [1], science began to deal with the field about 1695.

The applications of fractional order differentiation have attracted the attention of researchers from vide variety of science disciplines especially from the fields of applied sciences[2,3].

The fractional order PID has advanced rapidly in recent decades and has a wide range of applications in control engineering and research [4-8]. Pdlubny was the first to implement FOPID in 1997 [9]. He also showed how this form of controller responds more quickly than the classical PID controller, when its used for the control of fractional order systems.

It is well-known that to address the parametric uncertainty in both linear and non-linear systems, adaptive control method is one of the best control techniques being used so far. However, the focus was on the use of integer order systems to implement adaptive control method [10,11]. Monje and *al* [12] have reported the use of fractional calculus in conventional systems and control. The synchronization of chaotic systems with fractional order operator as well as the creation of an adaptive sliding-mode controller for such systems have both been described using adaptive function projective and feedback control techniques [13]. The effect of uncertain fractional order of chaotic systems can be controlled by adopting various practical methods such as an adaptive fractional-order switching-type control method, an adaptive fuzzy sliding-mode control method synchronization control method [11,13].

One of the most crucial methods in computing for locating exact or approximate answer to optimization is the employment of genetic algorithms.

Encoding, assessment, cross-over, mutation, and decoding are the essential components of the genetic algorithm process. The initial population is chosen at random, and each individual fitness is then calculated. The genetic algorithm's fitness function design is crucial since it has a significant impact on the output that is intended. Each person's fitness value is calculated by applying the fitness function [14].

The main contribution of this work is the use fractional adaptive PID controller approach optimized by genetic algorithm to improve the performances (rise time, setting time and overshoot) for two fractional systems by using the fractional order inside the traditional feedback adaptive PID controller. the optimizing parameters is obtained by using the fitness function.

The manuscript is organized as; firstly we have discussed the fundamentals of a fractional order system followed by the study of algorithms for integer and fractional adaptive PID controller. Afterwards, the results obtained with performance analysis from simulation applying for two systems o Fractional systems using the integer and fractional adaptive PID controller are presented. lastly, the conclusion along with future perspectives of the study is given.

Fractional Order Systems and Oustaloup's Approximation

Fractional calculus is a part of calculus theory which generalizes the derivative or integral of a function to non-integer order [15]. The number of applications where fractional calculus has been used grows rapidly mainly for the reason that these mathematical phenomena allow to describe a real object more accurately than the classical methods [16,17]. Approximation methods of fractional derivative and integral to rational functions permitted to use very easily fractional order systems in wide areas of applications such as control theory [18-24], economical systems [25] ,... etc.

The Oustaloup method is based on the function approximation from as;

(1) $$G_f(s) = S^{\alpha} \quad , \alpha \in R^{+}$$

By taking into account the rational function:

(2) $$G_f(s) = K \prod_{k=1}^{N} \frac{s + w_k'}{s + w_k}$$

However, the poles, zeros, and gain can be evaluated as;

$w_k' = w_b . w_u^{(2k-1-\gamma)/N}$, $w_k = w_b . w_u^{(2k-1+\gamma)/N}$, $K = w_h^{\gamma}$

Where w_u representsthe unity gain in frequency and the central frequency in a geometrically distributed frequency band. Let $w_u = \sqrt{w_h w_b}$, where, w_h and w_b represent the upper and lower frequencies, respectively. γ and N are the orders of derivative and filter, respectively.

Optimization through genetic algorithms

A genetic algorithm is a method for locating precise or approximate answers to optimization problems [26-27]. The essential steps in the genetic algorithm process are encoding, evaluation, cross-over, mutation, and decoding. The initial population is chosen at random, and each person's fitness is then calculated. The design of fitness function is particularly important in genetic algorithm Because the desired output significantly depends on the design of the fitness function

The steps of the algorithm of GA given in [28] are:

1. Choose the initial population
2. Each individual in the population should have their fitness evaluated.
3. Repeat

3.1. Choose the most qualified people to reproduce

3.2. Create a new generation through hybridization and mutation, then produce children.

3.3. evaluation of the fitness of the progeny.

3.4. Children should be used to replace the least desirable population.

4. Until termination

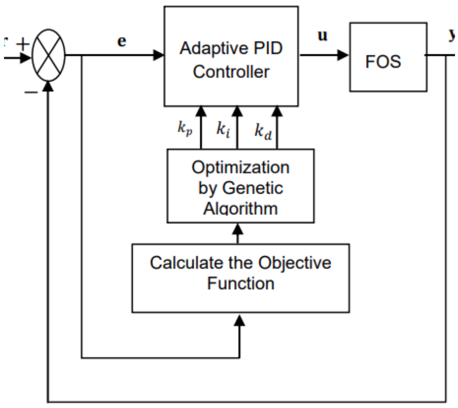

Fig. 1. Classical Adaptive PID Control of Fractional Order System (FOS).

Integer Adaptive PID Controller

The integer adaptive feedback control law is given by the equation 3, [12,23]:

(3) $$u(t) = -k_c[k_1(t)e(t) + I\{k_2(t)e(t)\} + D(k_3(t)e(t))]$$

With:

$k_1(t) = k_p(t) + \alpha_1 k_i(t) + \alpha_3 k_d(t)$

$k_2(t) = \alpha_2 k_i(t)$,$k_3(t) = \alpha_4 k_i(t)$, $k_p(t) = e^2(t)$

$k_i(t) = I\{e^2(t)\}$ and $k_d(t) = D\{e^2(t)\}$

$e(t) = y(t) - r(t)$

Where k_c, α_1 *and* α_2 are positive constants.

The ordinary schematic representation of the overall system is shown in figure 1.

We can see the extreme simplicity of the control system.

Fractional adaptive PI$^{\lambda}$D$^{\mu}$ Controller

The Fractional adaptive feedback control law is given by the equation (4):

(4) $$u(t) = -k_c\left[k_1(t)e(t) + I^{\lambda}\{k_2(t)e(t)\} + D^{\mu}(k_3(t)e(t))\right]$$

With:

$k_1(t) = k_p(t) + \alpha_1 k_i(t) + \alpha_3 k_d(t)$,$k_2(t) = \alpha_2 k_i(t)$, $k_3(t) = \alpha_4 k_i(t)$ and $k_p(t) = e^2(t)$, $k_i(t) = I^{\lambda}\{e^2(t)\}$,$k_d(t) = D^{\mu}\{e^2(t)\}$, $e(t) = y(t) - r(t)$.

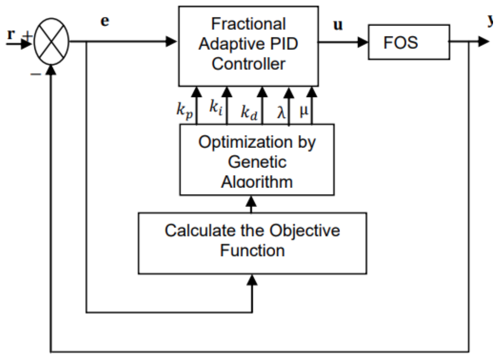

Fig. 2. Fractional Adaptive PID Control of Fractional Order System (FOS).

Results and Discussion

The error between the computed and measured output is used to determine the fitness function F that will be used for minimization. It is given by the Mean Absolute Error (MAE) [22,29] as follows:

(5) $$MAE = \frac{\sum_{i=1}^{N} y(i) - r(i)}{N}$$

were y is the measured output system and r is the desired output.

System 1

The Fractional order System is given by following transfer function:

(6) $$G(s) = \frac{10}{2.s^{1.4} + 5\ s^{0.5} + 1}$$

Figure 3 shows the output of the fractional system using the integer adaptive PID Controller with the following optimized parameters values:

$\boldsymbol{k_p = 412.8671, k_i = 222.2729, k_d = 49.1031}$

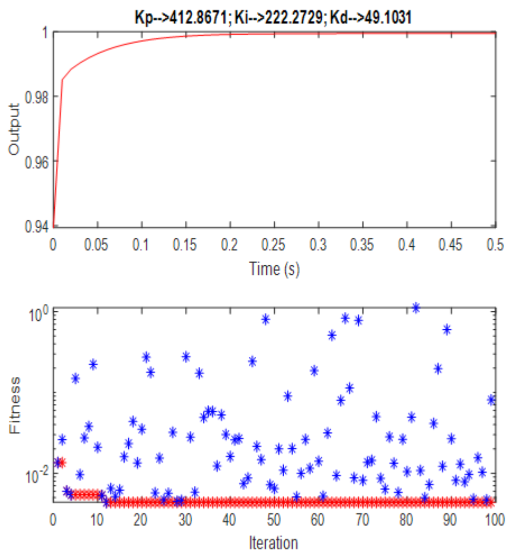

Fig. 3. Output of the fractional system using the Integer adaptive PID Controller.

Figure 4 shows the output of the fractional system using the Fractional adaptive PID Controller with the following optimized parameters values:
$\mathbf{k_p = 788.0445, k_i = 106.9445, k_d = 905.9804, \lambda = 0.47665, \ \mu = 0.88967}$

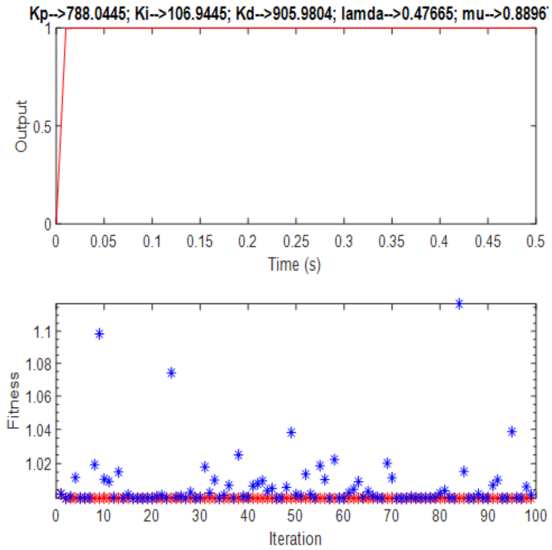

Fig. 4. Output of the fractional system using the Fractional adaptive PID Controller.

The performance analysis of the proposed fractional order adaptive PID controller and the classical adaptive PID controller is given by the following table:

We remark that the fractional adaptive PID Controller give the good improvement of setting time, rise time and mean absolute error comparatively to the Integer adaptive PID Controller results.

Table 1. Transient Response Stability Parameters of Aircraft System

Controller	Overshoot [%]	Setting time [s]	Rise time [s]	Mean Absolute Error (Rad)
APID	0.0000	0.1342	0 .0510	0.0072
FAPID	0.0000	0.0098	0.0080	0.0018

System 2

The second example is the heating furnace , Podlubny et al describes it in [9]. The Fractional order System is given by following transfer function:

(7) $$G(s) = \frac{1}{6484\, s^{1.0888} + 0.001\, s^{0.001} + 1.017}$$

Figure 5 shows the output of the fractional system using the integer adaptive PID Controller with the following optimized parameters values:
$\mathbf{k_p = 78114.5269, k_i = 19579.7981, k_d = 99235.8973}$

Figure 6 shows the output of the fractional system using the Fractional adaptive PID Controller with the following optimized parameters values:

$k_p = 8456.7123, k_i = 109574.1802, k_d = 188547.3969,$

$\lambda = 0.41774, \ \ \mu = 0.98305$

The performance analysis of the proposed fractional order adaptive PID controller and the classical adaptive PID controller is given by the following table 2.

We remark that the fractional adaptive PID Controller give the good improvement of setting time, rise time and mean absolute error comparatively to the Integer adaptive PID Controller results.

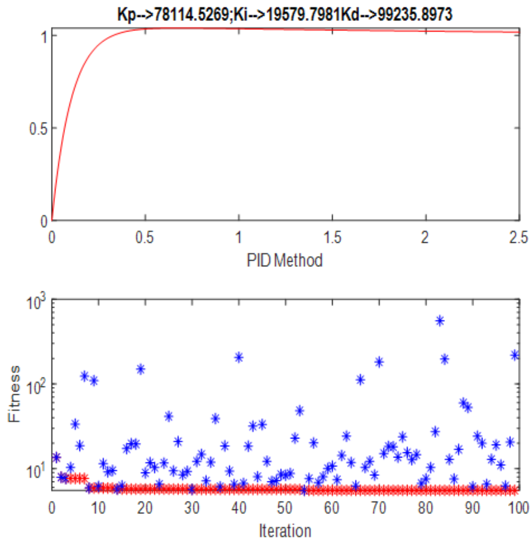

Fig. 5. Output of the fractional system using the Integer adaptive PID Controller.

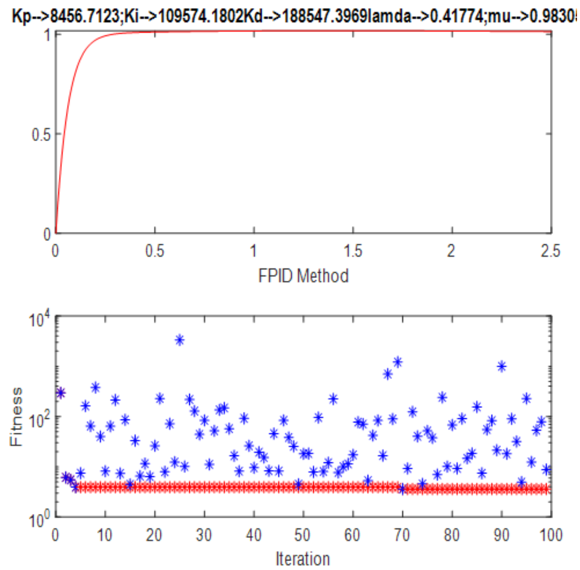

Fig. 6. Output of the fractional system using the Fractional adaptive PID Controller.

Table 2. Transient Response Stability Parameters of the heating furnace

Controller	Overshoot [%]	Setting Time [s]	Rise Time [s]	Mean Absolute Error
APID	2.1052	0.8216	0.2084	0.091
FAPID	0.8711	0.2152	0.1274	0.028

Conclusion

In this paper we study the performance analysis using the Integer adaptive PID Controller and the fractional adaptive PID Controller optimized by genetic algorithm applying to two Fractional systems . The Fractional approach allows to improve the good improvement of overshoot, setting time, rise time and mean absolute error comparatively to the Integer adaptive PID Controller results.

The simulation studies show the good performance of the proposed approach and confirm its superiority over Integer adaptive PID Controller. In future work, we will investigate the generalization of the fractional adaptive control approach to others fractional systems in order to improve their robustness and noise rejection.

Authors:
Mr. Asradj Zahir, University of Djelfa *Algeria, E-mail:* zahirasradj@yahoo.fr *Or* z.asradj@univ-bouira.dz *; Dr. Yassine Bensafia, Bouira University 10000, Algeria, E-mail:* bensafiay@yahoo.fr *(Corresponding author); Dr. Imad Merzouk,* University of Djelfa, *Algeria, E-mail:* i.merzouk@univ-djelfa.dz **.**

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