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Single-phase Cascade Inverter Controlled by Signals Calculated on the Basis of the Haar Wavelet

Streszczenie. *W artykule przedstawiono propozycję syntezy wartości chwilowej napięcia wyjściowego wielopoziomowego kaskadowego falownika napięcia. Opisano analityczną metodę wyznaczania zbioru falek ortogonalnych Haara oraz propozycję syntezy przebiegów wyjściowych falownika w oparciu o transformatę falkową. Na podstawie falki Haara obliczono sygnały sterujące kluczami połączonych kaskadowo dwupoziomowych falowników tworzących wielopoziomowy falownik napięcia. Przeprowadzono symulację współpracy takiego falownika z obciążeniem rezystancyjno-indukcyjnym oraz wykazano wpływ zmiany stałej czasowej na przebieg napięcia wyjściowego. (**Jednofazowy falownik kaskadowy sterowany wektorami obliczonymi na podstawie falki Haara**).*

Abstract. *The article presents a proposal for the synthesis of the instantaneous value of the output voltage of a multi-level cascade voltage inverter. An analytical method of determining the set of Haar orthogonal wavelets and a proposal for the synthesis of the output waveforms of the inverter based on the wavelet transform are described. On the basis of the Haar wavelet, signals controlling the keys of two-level inverters connected in cascade and forming a multi-level voltage inverter were calculated. A simulation of the cooperation of such an inverter with a resistive-inductive load was carried out and the influence of the change of the time constant on the course of the output voltage was demonstrated.*

Słowa kluczowe: falka Haara, transformata falkowa, falownik wielopoziomowy, synteza przebiegów falkowych.
Keywords: Haar wavelet, wavelet transform, multilevel converter, wavelet waveforms synthesis.

Introduction

In contemporary industry and public areas, devices like voltage and current converters are able to control and supply diverse equipment working in the power range of hundreds kW and more. These devices have to fulfil definite and diversified requirements which implies diversified purposes and methods of electric energy conversion. There are many industrial applications e.g. uninterruptible power supplies (UPS) or distributed power generation systems, where the essential demand is to generate 50 or 60 Hz sinusoidal voltage waveforms. The quality of generated waveforms, especially the Total Harmonic Distortion factor (THD), should comply with appropriate standards. In many devices like UPSs, active filters, or voltage regulators in electric energy grids, the main most important features are high-quality output waveforms, output stability, and efficiency of the device [1, 2, 3]. Similar requirements are to be fulfilled in converters applied in renewable energy systems.

The features, performance, drawbacks, and limitations of the two-level inverter have been largely recognized and verified in practice. Latest achievements in power semi-conductor technology permit it to work with higher frequency but fast switching accompanying the PWM control causes power losses in switching elements thus cutting inverter efficiency down [4].

Recently, multilevel inverters have emerged as a new and very important class of converters. Thanks to their promising performance, multilevel inverters are becoming more and more an alternative to conventional two-level inverters. They permit us to overcome the problem of limited power and shape output waveforms. As a result, many multilevel converters have been applied in the industry.

The development of multilevel converters comprises a novel area of research for new topologies, control strategies, and theory. Producing the required voltage or current waveforms is possible in many ways: e.g. sinusoidal PWM, selective harmonic elimination, space-vector modulation (SVM), or shaping the stepped voltage or current [5, 6]. Important works and studies concern the subject of frequency adjustment. Diverse methods of converters' control such as computing the adequate switching angles of stepped waveforms or cutting specified harmonics have been developed [6]. Using PWM methods provokes decreasing converter efficiency, which is a serious disadvantage, particularly in the range of higher-power applications.

The aforementioned disadvantages can be slightly reduced by the use of novel converters topologies as well as mathematical tools-aided control strategies. The paper deals with the technique of shaping the stepped output waveforms in multilevel converters. A mathematical approach to the control strategy based on wavelet transforms is presented. The output waveform synthesis is accomplished using a set of orthogonal wavelets. The discussion includes such mathematical tools as Haar wavelet transform [6,7]. The article presents the initial simulation studies of the developed single-phase inverter model controlled by wavelet signals. This is the first model that enables the implementation of the wavelet theory presented in [7]. The next step will be an attempt to implement the presented inverter and experimental research.

Wavelet waveforms synthesis

Wavelets is a term for mathematical functions, that allow the analysis of signals in different time scales and resolutions. The wavelets application are in many not directly related areas like seismology, video analysis, quantum mechanics, or electronics. The wavelets have been used mainly for analysing processes or signals based on the decomposition of the elements of the processes. The following considerations will prove that wavelets can be also useful in the composition of the power electronics signals and structures. The main application of the wavelet theory is used in control algorithms or in diagnostics [8] and the detection of various types of faults in converters, networks, or drives [9]. The Haar wavelet, on the other hand, is not so widely used.

The Haar wavelets have been adapted [6,7] for cascade inverter control. The Haar wavelet form is similar to the form of the voltage or current pulse that can be obtained using a simple one-phase inverter i.e. H-bridge cell. The displacement and width of the wavelet can be freely created and controlled. Thanks to these properties it is possible to apply wavelets in power electronics e.g. to form the output stepped waveforms of multilevel converters [10].

Let us define the scaling function $\varphi(x)$ in an interval $x\in<0,2\pi)$:

$$\varphi(x)=\begin{cases}1 & \text{for} \quad 0\le x<2\pi, \\ 0 & \text{for} \quad \text{other} \quad x\end{cases} \tag{1}$$

The fundamental proposed wavelet is defined:

$$\psi(x)=\begin{cases}1 & \text{for} \quad 0\le x<\pi, \\ -1 & \text{for} \quad \pi\le x<2\pi, \\ 0 & \text{for} \quad \text{other} \quad x\end{cases} \tag{2}$$

The wavelet determines one period of the rectangular wave and is the mother function introducing a family of wavelets:

$$\psi_{mn}(x)=\psi\left(2^{-m}x-2\pi n\right) \quad \text{for} \quad m,n=\ldots,-2,-1,0,1,2,\ldots \tag{3}$$

The wavelet scale is done as $2^m 2\pi$ and its displacement on the x-axis is determined as *n*-times $2^{m+1}\pi$. The *m* factor scales not only the wavelet but the amplitude too.

The scaling function and a few wavelets have been presented in Figure 1.

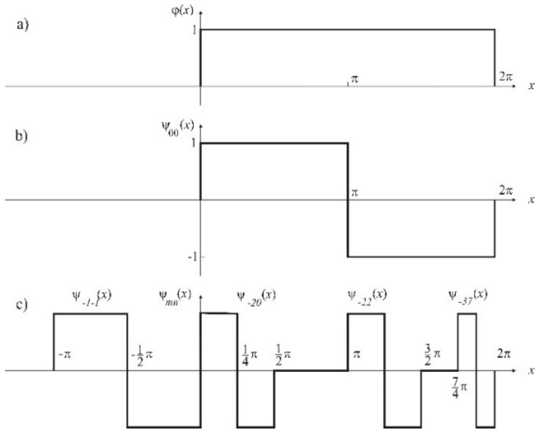

Fig.1. The scaling function $\varphi(x)$ and wavelets $\psi_{mn}(x)$: a) scaling function $\varphi(x)$, b) fundamental wavelet $\psi_{00}(x)$, c) wavelets $\psi_{-1-1}(x)$, $\psi_{-20}(x)$, $\psi_{-22}(x)$, $\psi_{-37}(x)$

All wavelets $\psi_{mn}(x)$ are orthogonal in the interval $x\in<0,2\pi)$. The defined statement (7) creates a family of orthogonal functions and can determine the basis of the wavelet transform. A continuous wavelet transform is defined as:

$$Wf(m,n)=\int_{-\infty}^{\infty} f(x)\,\psi_{mn}(x)\,\mathrm{d}x \tag{4}$$

and presents itself as a scalar product of a function *f*(*x*) and function $\psi_{mn}(x)$. The function f(x) reconstruction occurs when the inverse wavelet transform is applied (5).

In applications more comfortable is using a discrete inverse wavelet transform which is defined according to (6) by the equation:

$$f(x)=C\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}Wf(m,n)\,\psi_{mn}(x) \tag{5}$$

The symbol C denotes a constant that can be calculated from the Fourier transform of the function $\psi_{mn}(x)$. It could be written as a sum of wavelets $\psi_{mn}(x)$ multiplied by coefficients a_{mn}

$$f(x)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}a_{mn}\,\psi_{mn}(x) \tag{6}$$

The coefficients a_{mn}, called wavelet coefficients, are scalar products of the function *f*(*x*) and wavelets $\psi_{mn}(x)$. In the interval $x\in<0,2\pi)$ they are given as

$$a_{mn}=C_m\int_0^{2\pi} f(x)\,\psi_{mn}(x)\,\mathrm{d}x \tag{7}$$

The constant C_m is only dependent on coefficient *m* and has the same value for different *n*. Denoting

$$f_{mn}(x)=a_{mn}\,\psi_{mn}(x) \tag{8}$$

it is possible to write $f_\psi(x)$ as a sum of components $f_{mn}(x)$:

$$f_\psi(x)=\sum_{m=-3}^{m=0}\sum_{n=0}^{2^{-m}-1}f_{mn}(x) \tag{9}$$

The components are $f_{mn}(x)$ present component wavelets, amplitude, and phase which are determined by coefficients a_{mn}, calculated according to (7).

In power electronics, the most important criterion of the approximated waveforms is the THD factor. Practically in power electronics applications, the approximation of a sine wave should be realized using a finite number of wavelets. The natural aspiration of designers is to utilize the possibly lowest number of components. The accuracy of approximation depends on it. In mathematics, the accuracy is determined as an average square error δ, a very useful criterion destined for that purpose.

Let us denote by $f_\psi(x)$ a waveform approximating the function $f(x)=\sin(x)$ in the interval $x\in<0,2\pi)$. Assuming that $f_\psi(x)$ forms a combination of wavelets determined by index $m = -3, -2, -1, 0$ it can be written as a sum:

$$f_\psi(x)=\sum_{m=-3}^{m=0}\sum_{n=0}^{2^{-m}-1}a_{mn}\,\psi_{mn}(x)=\sum_{m=-3}^{m=0}\sum_{n=0}^{2^{-m}-1}f_{mn}(x) \tag{10}$$

All coefficients a_{mn}, calculated according to (7), have been collected in Table 1.

Table 1. The wavelet coefficients a_{mn}.

a_{mn}	n					
	0	1	2	3	4	5
a_{0n}	0.636	–	–	–	–	–
a_{-1n}	0	0	–	–	–	–
a_{-2n}	- 0.264	0.264	0.264	- 0.264	–	–
a_{-3n}	- 0.179	- 0.074	0.074	0.179	0.179	0.074

Successive steps of wavelet approximation $f_{\psi k}(x)$ for $k = 1, 2, 3$ have been presented in Figures 2, 3, and 4. The first step of reconstruction creates function $f_{\psi 1}(x)$ as a set of wavelets:

$$f_{\psi_1}(x)=\sum_{m=-2}^{m=0}\sum_{n=0}^{2^{-m}-1}f_{mn}(x)=f_{-20}+f_{-21}+f_{-22}+f_{-23}+f_{-10}+f_{-11}+f_{00} \tag{11}$$

in which two component wavelets are equal to zero according to Table 1. The waveforms are presented in Figure 2.

Figure 3 presents the result of the second step of approximation in which a few (but not all) wavelets f_{-3n} have been added to the function $f_{\psi 1}(x)$. Function $f_{\psi 2}(x)$ creates a composition of the following wavelets:

$$f_{\psi_2}(x)=f_{\psi_1}(x)+f_{-30}+f_{-33}+f_{-34}+f_{-37} \tag{12}$$

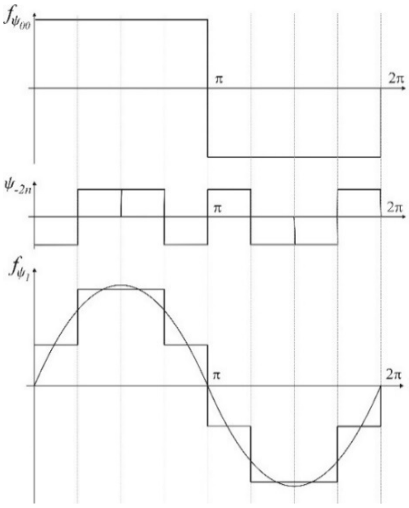

Fig 2. The first step of wavelet approximation:
$f_{\psi_1}(x) = f_{-20} + f_{-21} + f_{-22} + f_{-23} + f_{00}$

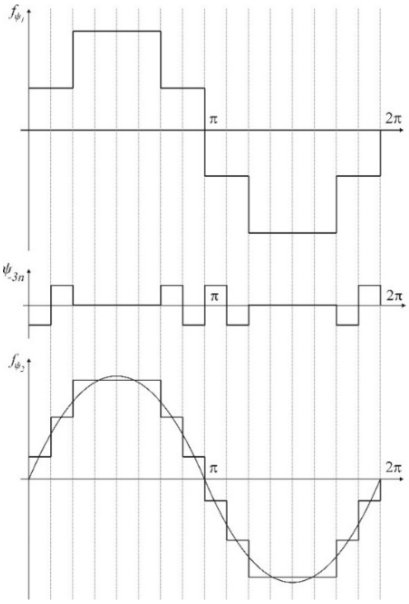

Fig 3. The second step of wavelet approximation:
$f_{\psi_2}(x) = f_{\psi 1} + f_{-30} + f_{-33} + f_{-34} + f_{-37}$

Wavelet converter

One-phase wavelet voltage converter consists of three voltage inverters (respectively transistors T1÷T4, T5÷T8, T9÷T12,) connected in cascade. The proposed method based on wavelet transform is very suitable for cascade converters even though it demands independent voltage sources. In each phase of the complex converter, the component inverters are supplied from three independent voltage sources: U_{D1}, U_{D2}, and U_{D3}. The supply voltages are proportional to the relevant amplitudes of component wavelets. If the proportional factor is set to 500 the voltage U_{D1} = 318 V, U_{D2} = 132 V, and U_{D3} = 84 V.

The control of such a converter is derived from described wavelets model. The inverter control signals, which were calculated from the Haar wavelet, are shown in Figure 4. The schematic diagram of the one-phase converter is presented in Figure 5. The first inverter, counting from the top, generates the output voltage with the waveform $f_{\Psi 00}$, the second with the waveform ψ_{-2n}, and the third with the waveform ψ_{-3n}.

The summing process of the wavelets in the cascade is accomplished by serially connecting the outputs of the component inverters whose switching frequencies differ. Assuming that the fundamental frequency is 50 Hz, the frequencies of individual wavelets are different and take the following values: for $f_{\Psi 00}$ is f = 50 Hz, for ψ_{-2n} is f = 200 Hz, for ψ_{-3n} is f = 400 Hz. For example, for wavelet ψ_{-2n} due to the phase change, there will appear pulses of 5 ms duration two times in each output voltage period, and for wavelet ψ_{-3n} appear pulses of 2.5 ms duration.

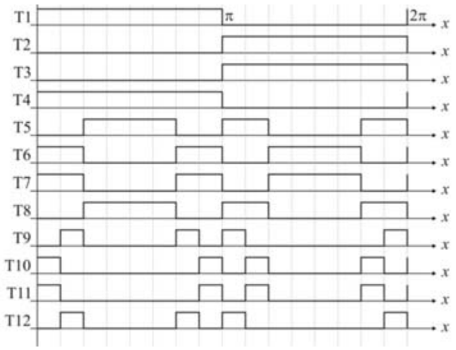

Fig. 4. Control ranges of individual transistors in a single-phase six-level voltage inverter system realizing the wavelet model of the converter

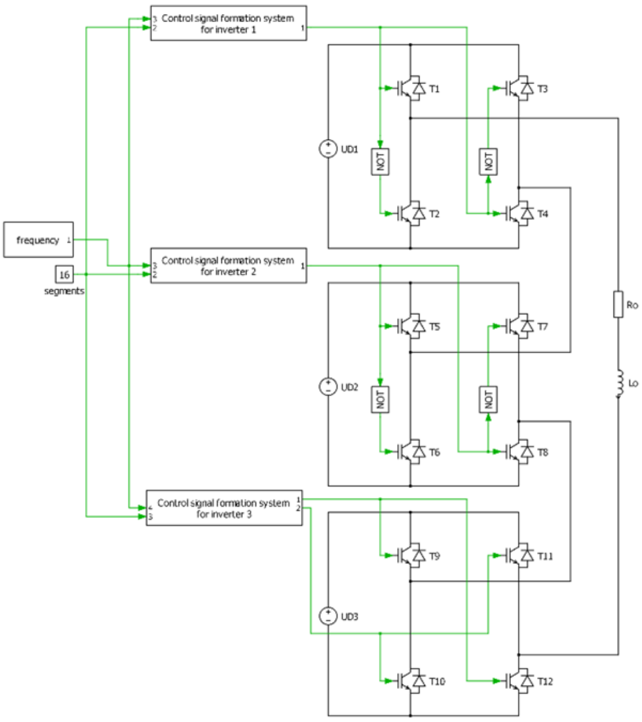

Fig. 5. Scheme of a cascade inverter with signal forming systems

Simulation tests were carried out in the PLECS program and the RT Box device, which enables HIL (hardware in the loop) simulations. In simulation tests, the cooperation of the proposed inverter with resistive and resistive-inductive loads was checked. With a resistive load, the output voltage and current are similar in shape, so they are not included as a result of the simulation. A more interesting situation occurs with resistive-inductive loads.

Figure 6 shows examples of voltages and current waveforms at the load R_0 = 5 Ω and L_0 = 20 mH and for the fundamental frequency *f* = 50 Hz.

For the load Ro = 5 Ω, Lo = 20 mH, the inverter works properly and the time constant calculated as $\tau = Lo/Ro$ is $\tau = 4$ ms. After increasing the inductance in the load to 35 mH, the time constant increased to 7 ms. Such an increase in the time constant causes significant disturbances in the operation of the third inverter, which is shown in Figure 8.

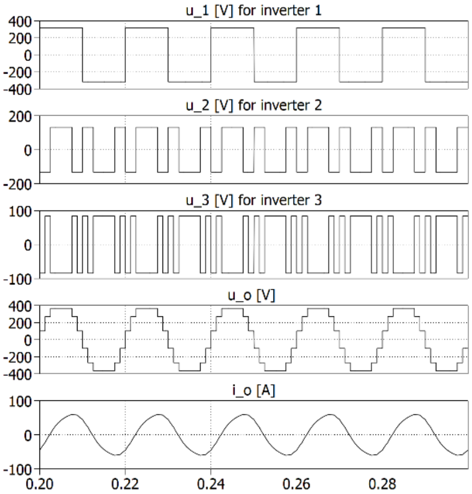

Fig. 6. Waveforms of voltages and current for individual inverters and loads: Ro = 5 Ω and Lo = 20 mH

Figure 7 shows the spectra for waveforms from Figure 6.

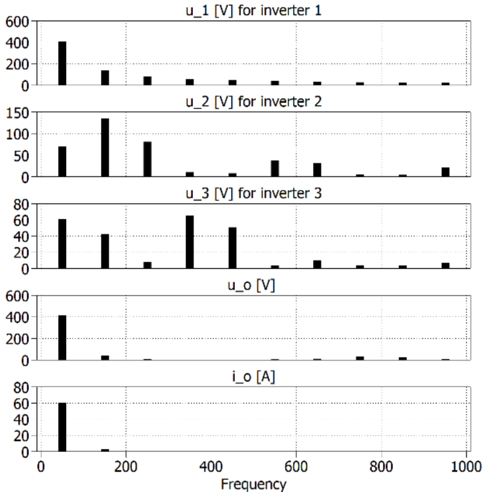

Fig. 7. Spectra of voltages and current for individual inverters and loads Ro = 5 Ω and Lo = 20 mH

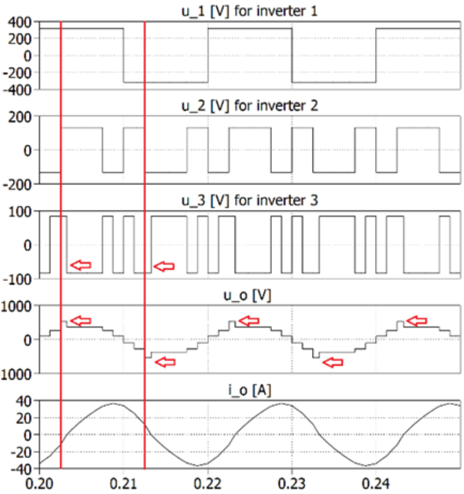

Fig. 8. Waveforms of voltages and current for individual inverters and loads: Ro = 5 Ω and L = 35 mH

Increasing the load time constant modifies the switching times of the third inverter and the appearance of ripples in the output voltage waveform. Ripples are symmetrical in the output voltage waveform, therefore they do not cause the occurrence of a DC component in the output voltage waveform, which in some applications would be a serious disadvantage of this inverter.

Additional tests carried out have shown that the cascade inverter consisting of two single-phase inverters is not sensitive to changes in the resistive-inductive load, as it is in the case of a cascade of three single-phase inverters. However, the use of a third inverter in the cascade significantly improves the THD of the output voltage. In the case of two inverters in a cascade, the output voltage has THD= 24% and with three inverters already THD= 18%.

Conclusion

The cascade inverter controlled by signals calculated on the basis of the Haar wavelet allows obtaining a waveform of the output voltage with a very low content of THD without using PWM modulation and additional filters. On the other hand, an RL load with a low inductance value, allows to obtain a current waveform similar to a sine wave with a low THD = 4,4 %. As a result, the proposed system with wavelet control is an interesting proposition in the case of intensively developing systems with multi-level inverters in renewable energy sources. Unfortunately, as simulation tests have shown, the cascade inverter, which consists of a larger number of inverters in the cascade, is very sensitive to the increase in the time constant, which, with larger values, changes the switching times of transistors and deteriorates the quality of voltage and current waveforms.

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LITERATURE

[1] Chiasson J. N., Tolbert L. M., McKenzie K. J., Zhong Du.: Control of a Multilevel Using Resultant Theory. IEEE Transactions on Control Systems Technology, (2003), vol. 11, no. 3,

[2] Daubechies I.: The wavelet Transform, time-frequency localization and signal analysis. IEEE Transactions on Informatics Theory, (1990), vol. 36, pp. 961-1005

[3] Faranda R., Valade I.: UPQC Compensation Strategy and Design Aimed at Reducing Losses. IEEE International Symposium on Industrial Electronics ISIE (2002), vol. 4, pp. 1264-1270

[4] Graps A.: An Introduction to Wavelets, IEEE Computa-tional Science and Engineering, (1995), vol. 2, no. 2.

[5] Haar A.: Zur Theorie der orthogonalen Funktionensysteme, Mathematische Annalen, 1910, Vol. 69, pp. 331-371

[6] Muc A., Iwaszkiewicz J., Active Filtering of Inverter Output Waveforms Based on Orthogonal Space Vector Theory, Energies 2022, 15(21), 7861, https://doi.org/10.3390/en15217861

[7] Iwaszkiewicz J., Perz J. – „A Novel Approach to Control of Multilevel Converter Using Wavelets Transform", RE&PQJ, Vol. 1, No.5, March 2007, https://doi.org/10.24084/repqj05.371

[8] Saleh S. A., Balancing Capacitor Voltages in 7-Level Single Phase Flying-Capacitor Wavelet Modulated Inverters, 2022 IEEE Industry Applications Society Annual Meeting (IAS), (2022), 10.1109/IAS54023.2022.9939910

[9] Eddine Ch. B. D., Azzeddine B., Mokhtar B., Detection of a two-level inverter open-circuit fault using the discrete wavelet transforms technique, 2018 IEEE International Conference on Industrial Technology (ICIT), (2018), 10.1109/ICIT.2018.8352206

[10] Eswar K. N. D. V. S., Doss M. A. N., Vishnuram P., Selim A., Bajaj M., Kotb H., Kamel K., Comprehensive Study on Reduced DC Source Count: Multilevel Inverters and Its Design Topologies, Energies, (2023), 16, 18. https://doi.org/10.3390/en16010018