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doi:10.15199/48.2025.03.05

Memristor-based adaptive leaky integrate-and-fire neuron model: a simulation study

***Abstract**. Due to the increasing demand for AI algorithms and the limitations of traditional von Neumann architectures, there is a need to develop new, more efficient architectures that operate in analogy to the human brain. In this study, an attempt was made to replicate the behaviour of a human neuron using memristor-based circuits. The correctness of the proposed solutions was verified through computer simulations.*

Streszczenie.** Ze względu na rosnące zapotrzebowanie na algorytmy sztucznej inteligencji oraz ograniczenia wynikające z tradycyjnych architektur von Neumanna, istnieje konieczność opracowania nowych, bardziej efektywnych architektur, które będą działać w analogii do funkcjonowania ludzkiego mózgu. W niniejszej pracy podjęto próbę odwzorowania zachowania ludzkiego neuronu, wykorzystując układy oparte na memrystorach. Poprawność zaproponowanych rozwiązań została zweryfikowana za pomocą symulacji komputerowych. **(Oparty na memrystorze model neuronu Adaptive Leaky Integrate-and-Fire: Badania symulacyjne)

Keywords: memristor, neuromorphic computing, leaky-integrate-and-fire neuron
Słowa kluczowe: memrystor, obliczenia neuromorficzne, model neuronu LIF

Introduction

Conventional computers, based on the von Neumann architecture, face limitations due to the separation of processing and memory units, resulting in the von Neumann bottleneck. To overcome this issue, neuromorphic computing has been proposed, promising faster computing speeds and more energy-efficient systems [1]. The leaky integrate-and-fire (LIF) spiking model effectively mimics the firing patterns and information propagation of biological neurons, making it a valuable tool in neural networks, cognitive computing, and brain-inspired computing [2]. The Adaptive Leaky Integrate-and-Fire model is superior to the standard LIF model due to its additional adaptation variable and exponential voltage dependence. This allows it to accurately emulate a wider range of neuronal firing patterns, such as adapting, bursting, delayed spike initiation, initial bursting, fast spiking, and regular spiking, providing a more comprehensive and flexible model for neuronal behaviour [3]. In this paper, an Adaptive LIF neuron model based on a memristor is proposed, which will simplify circuits and increase the density of packing them into artificial neural network circuits directly in the hardware. The controlled resistance of the internal memristor allows the neural function to be regulated by controlling the current leakage rate.

Materials and Methods

For the simulation of circuit, the `LTSpice` and `Matlab` simulation environments were utilized, employing the Runge-Kutta algorithm, specifically RK4, to solve differential equations. To model the dynamics of memristors, the mean metastable switch (MMS) model was applied, as it accurately replicates the dynamics of self-directed channel (SDC) memristors [4, 5, 6], as described in the publications [7, 8, 9]. The parameters of the MMS models were optimized by fitting them to real measurement data.

LIF Neuron Model

Hodgkin and Huxley proposed the membrane electrical circuit to mimic the electrophysiological behaviors of the biological cell membrane based on their experiments on the squid giant axon [1]. The LIF spiking neural model simplifies the complex Hodgkin-Huxley (HH) model, making it easier to construct large-scale neural networks while maintaining biological plausibility [10, 11]. The cell membrane, consisting of a lipid bilayer (capacitor) and ionic channels (resistor), can be modeled using an electrical circuit. The LIF model, which its electrical circuit is shown in the Figure 1, includes external stimulus i_{in}, membrane capacitance C_n, resistance R_n, resting voltage v_{rest}, resistive voltage $v_c - v_{\mathrm{rest}}$, and currents passing through the capacitor i_c and resistor i_R [12]. When the membrane potential v_c exceeds the threshold voltage v_{th}, then capacitor is discharged and output spike is generated. The operational principle of the Leaky Integrate-and-Fire (LIF) neuron and its fundamental assumptions are illustrated in Figure 2. The dynamic of the LIF neuron model is described by the state Eq. (1).

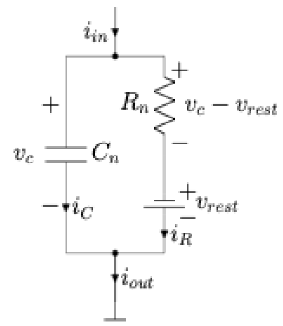


Fig. 1. Electric circuit model for LIF neuron model

$$\frac{dv_c}{dt} = \frac{1}{C} i_{in} - \frac{v_c}{RC} - \frac{v_{\mathrm{rest}}}{RC} \quad (1)$$

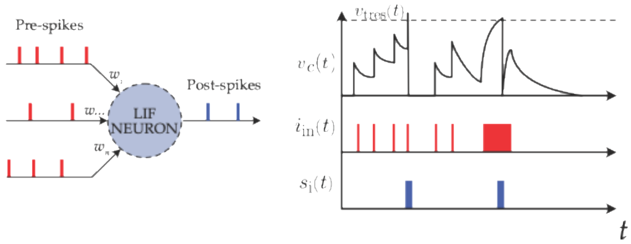


Fig. 2. The operational principle of the Leaky Integrate-and-Fire (LIF) neuron

Adaptive LIF neuron model

Described LIF model, which dynamics is described by single equation is, however, not sufficient to describe the variety of firing patterns that neurons exhibit in response to a step current. It doesn't put under consideration, its threshold adaptation based on the input constant current, and therefore spike generation (bursting) frequency [1,13], that could be defined as: $f_s = \frac{1}{\mathrm{ISI}}$ where ISI is the inverse interspike interval $\mathrm{ISI} = \Delta + \tau_m \log\left(\frac{i_{\mathrm{in}}R_n + v_{\mathrm{rest}}}{i_{\mathrm{in}}R_n + v_{\mathrm{rest}} - v_{th}}\right)$ where Δ is asymptote given by absolute refractory period which is the interval of time during which a second action potential cannot be initiated [14]. The operational principle of the adaptability was shown in Figure 3. In the proposed solution, the voltage across the memristor is analogous to the membrane potential. The memristor-resistor configuration forms a dynamic voltage divider, which allows for the adaptation of the bursting frequency to high-frequency input signals. As the current through the memristor increases, the value of its internal state variable x_m also increases, which subsequently decreases its resistance. This decrease in resistance leads to a reduction in the voltage across the memristor. Thus, assuming a constant threshold voltage v_{th}, this mechanism facilitates the adaptation to the input signal. Over time, the residual voltage v_{rest} causes a reset of the state variable x_m to zero. Hence, it was decided to propose a new memristor-based adaptive LIF model based on the [15]. The proposed schematic diagram of the adaptive LIF neuron model is shown in Figure 4a and the proposal of the hardware implementation is shown in Figure 4b. The proposed hardware architecture of the analysed Adaptive Leaky Integrate-and-Fire (LIF) Model is based on a memristor and incorporates several key components. These include a differential amplifier, which is implemented using the operational amplifier OP482, and a Schmitt trigger comparator, which utilizes the comparator model AD856. The hysteresis loop of the Schmitt trigger comparator has been carefully adjusted to ensure that it deactivates the MOSFET responsible for discharging the capacitor at the lowest possible voltage levels.

The dynamic of the proposed model is described by the system of algebraic-differential equations placed in Eq. (2).

$$
\begin{cases} \frac{\mathrm{d}v_C}{\mathrm{d}t} = \frac{1}{C}\left(i_{in} - \frac{v_C - v_{\mathrm{rest}}}{R_m(x_m) + R_s}\right) \\ \frac{\mathrm{d}x_m}{\mathrm{d}t} = f\big(x_m(t), i_m(t)\big) \end{cases} \tag{2}
$$

Where x_m represents the internal state variable of the memristor, i_m denotes the current passing through the memristor, and R_m is the memristor's resistance, which is a function of its internal state variable. The evolution of the internal state variable x_m is governed by a dynamic function $f(x_m, i_m)$, which encapsulates the relationship between the state variable and the current.

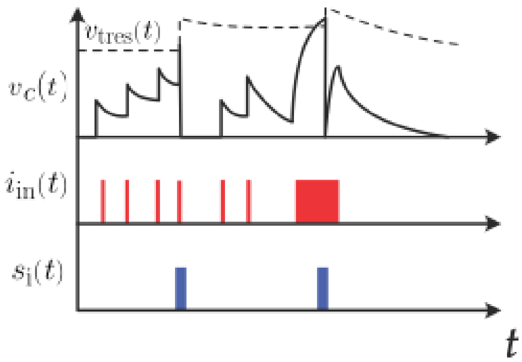
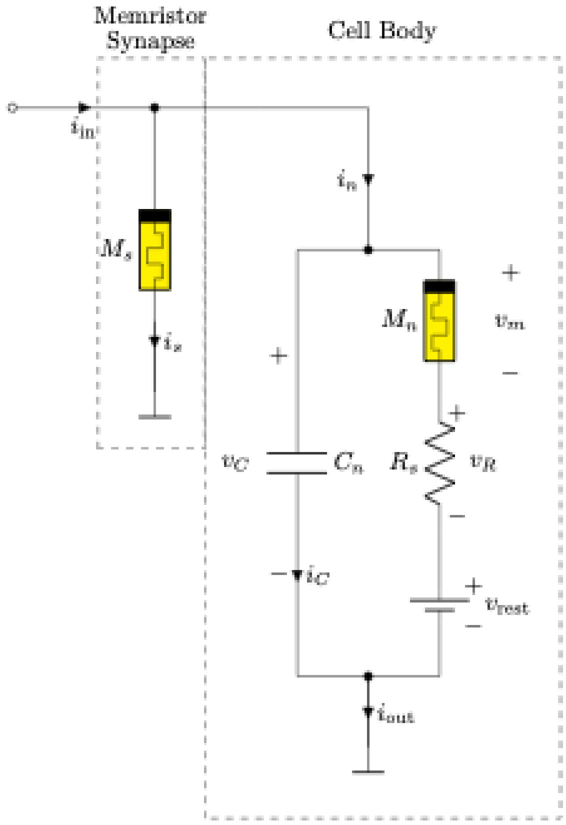


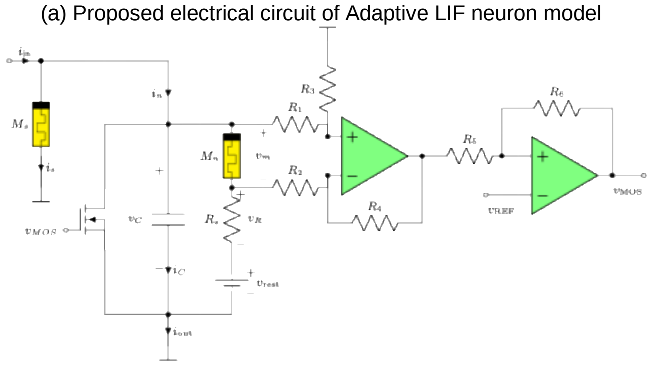
Fig. 3. The operational principle of the Adaptive Leaky Integrate-and-Fire (LIF) neuron

In order to best represent the real physical memristors the mean metastable switch (MMS) memristor model was used in the simulations [16]. The parameters of this model were matched to the actual measurement data of SDC memristors through numerical optimization.

The considered neuron models were implemented both in the `Matlab` environment, where the system of differential equations was solved using the Runge-Kutta method, specifically RK4, and in the `LTSpice` environment, where a circuit for possible physical implementation of the discussed neuron was proposed. Example simulation results are presented in Figure 5 and 6.



(a) Proposed electrical circuit of Adaptive LIF neuron model



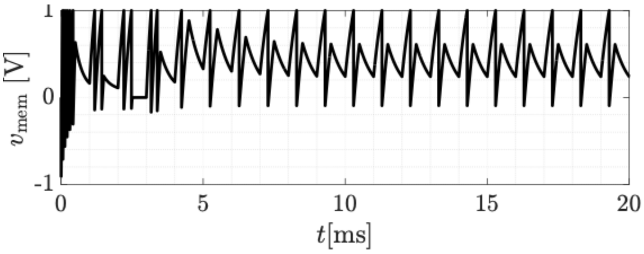
(b) Proposed hardware implementation of Adaptive LIF neuron model

Fig. 4. Proposed electrical schematics for the potential implementation of the Adaptive Leaky Integrate-and-Fire (LIF) Neuron.

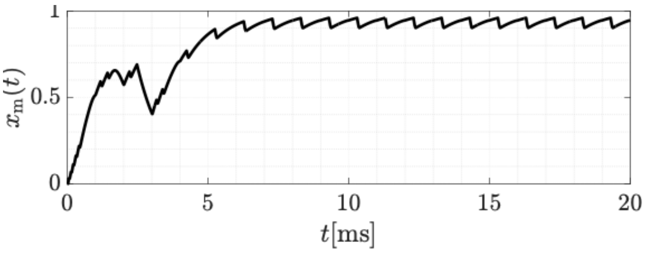
Results

In this section, we present the findings derived from the simulations and experimental evaluations conducted on the proposed Adaptive Leaky Integrate-and-Fire (LIF) Model based on a memristor

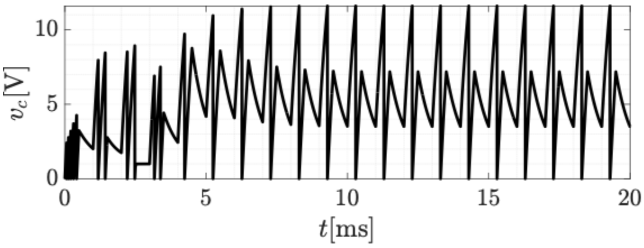
In Figure 5, sample simulation results for the memristor-based Adaptive Leaky Integrate-and-Fire (ALIF) model are shown, which solve simple differential equations implemented in the `Matlab` environment. Figure 5a depicts the time evolution of the voltage across the memristor. As predicted, during the initial moments of interaction with the forcing current, there were rapid voltage spikes, resulting in a high frequency of generated spikes. Subsequently, as the memristor transitioned into a low-resistance state, this frequency decreased. This behaviour demonstrates the adaptive nature of the neuron. Figure 5b shows the time evolution of the internal state variable of the memristor x_m, during the test. It can be observed that x_m increases with the excitation, which in turn decreases the resistance of the memristor. This mechanism enables the memristor to adapt to high excitation levels. Figure 5c presents the voltage across the capacitor, v_c, which mimics the membrane potential. However, in this case, the capacitor primarily serves to store charge, while the membrane potential is simulated by $\mathrm{v}_{\mathrm{mem}}$. It is noticeable that as the resistance decreases, a higher voltage is required to charge the capacitor, necessitating a greater amount of charge.



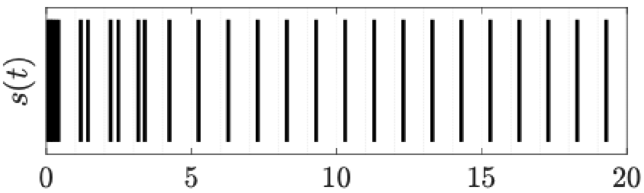
(a) Time series of the voltage across memristor



(b) Time series of internal state variable x_m at all simulation time.



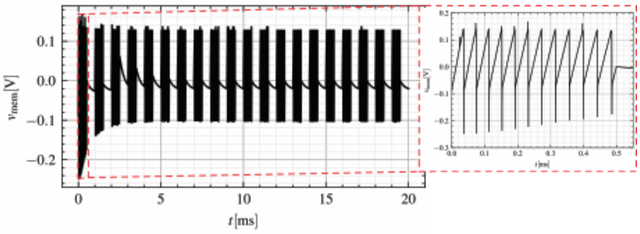
(c) Time series of the voltage across capacitor. It simulates membrane potential



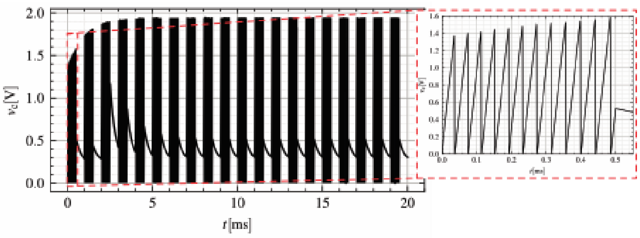
(d) Time series of generated spikes.

Fig. 5. The simulation results for the Adaptive Leaky Integrate-and-Fire (ALIF) Neuron in the `Matlab` environment, by solving ODE equations. The input current $i_{\mathrm{in}}$was a unipolar square wave with a frequency of $f = 1\mathrm{kHz}$, a duty cycle of $T = 50\%$, and amplitude $I_{\mathrm{amp}} = 1\mathrm{mA}$.

Figure 6 shows the simulation of the proposed Adaptive Leaky Integrate-and-Fire (ALIF) circuit, which can be directly implemented in hardware. Figure 6a shows the time evolution of the voltage across the memristor, v_{mem}. Here, the adaptation effect, similar to the one observed in Figure 5a, is also noticeable, though not as pronounced. This is likely due to the damping effect of auxiliary components such as operational amplifiers and comparators. A similar situation is observed in Figure 6b, which shows the voltage across the capacitor. The neuron's adaptation is visible, but not as pronounced as in the previous example. It appears that the causes are similar, likely due to the damping effects of auxiliary components such as operational amplifiers and comparators. Figure 5d displays the generated spike signal as a digital signal. Compared to the previous case, this plot also does not clearly demonstrate the adaptive nature of the LIF neuron. Figure 6d shows the time evolution of the internal state variable of the memristor during the test. As predicted, throughout the test, the memristor tended towards a low-resistance state R_{ON} by increasing the value of the state variable.

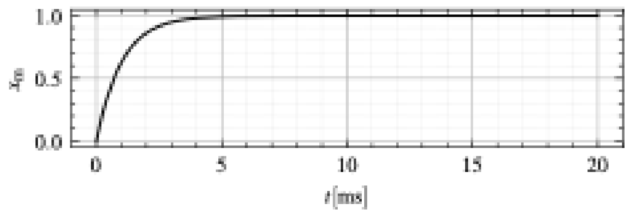


(a) Time series of the voltage across memristor



(b) Time series of the voltage across capacitor. It simulates membrane potential

(c) Time series of generated spikes at shorter time horizons to better show signal variability



(d) Time series of internal state variable x_m at all simulation time.

Fig. 6. The simulation results for the hardware implementation of the Adaptive Leaky Integrate-and-Fire (ALIF) Neuron in the `LTSpice` environment. The input current $i_{\mathrm{in}}$was a unipolar square wave with a frequency of $f = 1\mathrm{kHz}$, a duty cycle of $T = 50\%$, and amplitude $I_{\mathrm{amp}} = 1\mathrm{mA}$.

Conclusions

In summary, the proposed electronic circuit based on the Adaptive Leaky Integrate-and-Fire (LIF) model promises robust performance by more accurately mimicking the behavior of a human neuron compared to conventional LIF models. This is achieved through the unique nonlinear characteristics of memristors, which confer the neuron with the ability to adapt to specific stimuli, a highly desirable trait for neuromorphic computing. Constructing such a circuit allows it to be utilized as one of the processing stages directly on edge devices, effectively creating a deep layer for analog signal processing.

The Adaptive LIF model's primary advantage lies in its dynamic response to varying input signals. Memristors, with their state-dependent resistance, facilitate this adaptability by adjusting their resistance based on the history of voltage and current. This property enables the circuit to modify its behavior in real-time, providing a closer approximation to the biological processes occurring in human neurons. This adaptability is particularly beneficial for applications requiring real-time data processing and decision-making, such as robotics, autonomous systems, and advanced sensor networks.

The proposed circuit's ability to adjust its response based on the input signal's frequency and amplitude underscores its potential for efficient energy usage. In neuromorphic systems, managing power consumption while maintaining high performance is crucial. The memristor-based Adaptive LIF model achieves this balance by reducing the frequency of spikes as the memristor transitions to a low-resistance state. This reduction in spiking frequency corresponds to lower energy expenditure, making the system more sustainable for long-term operations.

Additionally, the inherent simplicity of the circuit design, relying on memristors, operational amplifiers, and comparators, makes it suitable for integration into existing hardware platforms. This compatibility ensures that the Adaptive LIF model can be adopted without extensive modifications to current systems, facilitating a smoother transition towards more advanced neuromorphic architectures.

Future research will focus on several key areas to advance the development and application of this circuit. First, empirical validation through extensive testing will be essential to compare real-world performance with simulation outcomes. This will involve building and testing multiple prototypes to assess consistency and reliability across different scenarios. Additionally, exploring the integration of this circuit into larger neuromorphic networks will be crucial for understanding its scalability and interaction with other components.

Future research will also focus on integrating neuron models with currently known machine learning methods. The tests will involve solving practical AI problems using hybrid models, where one of the processing stages will be the memristor-based LIF models. The accuracy of the results will be investigated and compared with classical machine learning methods.

In conclusion, the development of the Adaptive LIF model based on memristors represents a significant step forward in neuromorphic engineering. By providing a more accurate emulation of neuronal behavior and offering adaptability to varying stimuli, this model holds promise for a wide range of applications in artificial intelligence and edge computing. The continued research and refinement of this technology will pave the way for the creation of more advanced and capable neuromorphic systems, bringing us closer to achieving brain-like computation in hardware.

Overall, the memristor-based Adaptive LIF model not only enhances the fidelity of neuromorphic circuits but also opens new avenues for energy-efficient and adaptive computation. As we move towards more complex and autonomous systems, the importance of such adaptive mechanisms cannot be overstated. This work lays the foundation for future advancements in neuromorphic computing, where the integration of biological principles into electronic systems will drive the next generation of intelligent devices.

Acknowledgment: The research presented in the article was financed by the Polish Ministry of Science and Higher Education as part of a subsidy for AGH University of Krakow.

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