1. Piotr DERUGO, 2. Amanuel HAFTU KAHSAY, 3. Krzysztof SZABAT, 4. Rafał ZAWIŚLAK, 5. Kosuke SHIKATA, 6. Seiichiro KATSURA

ORCID: 1. 0000-0001-7030-6118; 2. 0009-0007-1540-6168 3. 0000-0001-8473-9475 4. 0000-0002-8513-8679 5. 0009-0007-2041-6168 6. 0000-0002-7487-0610

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Application of higher-order disturbance observer in the control structure of the two-mass system

Zastosowanie obserwatora zakłóceń wysokiego rzędu w strukturze sterowania układu dwumasowego

Abstract: *The article presents issues related to vibration damping in a dual-mass system. A control structure with a PI controller and with additional feedback from the torsional torque and its derivative is used for the study. A higher-order integral estimator is proposed to obtain information on these variables. Results showing a significant improvement in the quality of estimation of these quantities compared to the classical observer are presented. Theoretical considerations are confirmed by simulation and experimental studies.*

Streszczenie: *W artykule przedstawiono zagadnienia związane z tłumieniem drgań w układzie dwumasowym. Do badań wykorzystano strukturę sterowania z regulatorem PI i z dodatkowym sprzężeniem zwrotnym od momentu skrętnego i jego pochodnej. W celu uzyskania informacji o tych zmiennych zaproponowano estymator całkowy wyższego rzędu. Przedstawiono wyniki ukazujących znaczną poprawę jakości estymacji tych wielkości w porównaniu z klasycznym obserwatorem. Rozważania teoretyczne potwierdzono przez badania symulacyjne i eksperymentalne.*

Keywords: two-mass system, vibration damping, state estimation, electrical drives
Słowa kluczowe: układ dwumasowy, tłumienie drgań, estymacja zmiennych, napęd elektryczny

Introduction

When implementing advanced control structures, it is necessary to know the state variables of the drive system [1] - [7] . These variables are speed, position, load, current and others. Some of them are easy and cheap to obtain, others, on the contrary. The speed of the drive can be obtained easily using an encoder or resolver. However, the measurement of torques can be complicated (torsional) or sometimes even impossible (load). Currently, standard drive systems very often rely only on voltage and current sensors, leaving the others aside. A flexible coupling drive system is usually assumed to have a speed sensor [9] - [18] . For better control quality, various feedback can be used, especially torsional torque and its derivative.

Since this variable is hard to obtain, it can be estimated. Several methods for estimating state variables are mentioned in the literature. Among the simplest of these is the disturbance observer [12] . It determines the torsional torque based on the driving motor torque and speed. In its classical form, it is sensitive to measurement noise, especially in the speed signal. To reproduce the entire state vector, a Luenberger observer is very often used [18] [19] . This system is characterised by its relative simplicity of design. The fundamental problem is the correct choice of correction coefficients to ensure that the poles of the closed system are appropriately localised. A sliding observer is recommended for a system in which the parameters are not precisely known or there are other disturbances [18] . It provides better quality estimation of the state variables compared to the classical solution. For systems with a high level of measurement noise, a Kalman filter is recommended [20] . Information on state variables is necessary to implement advanced control structures [1] - [6] For a dual-mass system, these are usually based on unavailable torque and speed signals. In practical applications, the torsional torque and load speed are provided by means of estimators.

The paper [13] presents a new control structure with a PI controller and with additional couplings from the torsional torque and its derivative. It provides an arbitrary pole location of the closed-loop system. It thus represents an interesting alternative to the standard control structure, also based on feedback from the speed of the working machine. This structure is considered in the paper.

The main goal of the paper is to design a couple of high-order disturbance observers. Classical disturbance observers are sensitive to measurement noise. For this reason, integral observers are designed in this work. Analytical relationships are derived that allow for an arbitrary distribution of the observer poles. The influence of design parameters on the quality of variable estimation by observers is analysed. Simulation studies are confirmed by laboratory bench tests. In the paper, this kind of observer is also used, but tuning is performed with the help of a metaheuristic algorithm.

In this paper, we compare the usefulness of such a high-order observer for disturbance torque and its first derivative estimation with the more basic structure of an observer for disturbance torque estimation and the classical approach. The simulation results are backed by experimental verification. The pole placement equations for the considered systems are presented along with diagrams of the systems. A numerical simulation comparison is presented. Both simulation and experiments show smaller noise and better accuracy in obtained estimated variable value.

Mathematical model of the two-mass system and the control structure

The subject of the study is a drive system with an elastic connection. It consists of concentrated masses of motor and load distributed at the ends of an elastic shaft. A model of a mechanical system with an inertial elastic connection, commonly used in many works in this field [9] - [13] , is adopted for consideration. The object under study is described by the following equations (in relative units):

$$\dot{\omega}_1 = \frac{1}{T_1}\left(m_e - m_s\right) \quad (1)$$

$$\dot{\omega}_2 = \frac{1}{T_2}\left(m_s - m_L\right) \quad (2)$$

$$\dot{m}_s = \frac{1}{T_c}\left(\omega_1 - \omega_2\right) \quad (3)$$

where: ω_1, ω_2 – the speeds of motor and load side respectively, m_e, m_s, m_L – the electromagnetic, torsional, and load torques, T_1, T_2 – the mechanical time constant of the motor and load side, T_c – the parameter which represents the elasticity of the coupling

In Fig. 1, the block diagram of the dual-mass system is presented:

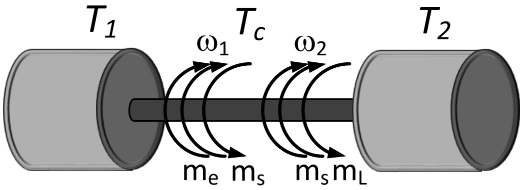

Fig. 1. Block diagram of the two-mass system

A schematic diagram of the control system is demonstrated in Fig. 2. It consists of the following parts: a two-mass system, a driving torque control loop, a PI-type speed controller, and additional feedback from the torsional torque (k_1) and its derivative (k_4). The designation of additional couplings is [17] .

The RRC's control structure is built on the same coupling from the torsion torque [12] . However, it is based on a different approach to obtaining the correct value of the resonant to anti-resonant frequency. In order to place the poles of the circuit at arbitrary locations and thus obtain arbitrary variable waveforms in the linear operating range, it is necessary to introduce two additional couplings from different quantities into the circuit with the PI controller. As can be seen from [17] , couplings from two other groups should be used. Particularly popular is a system with additional coupling from torsional torque and speed difference.

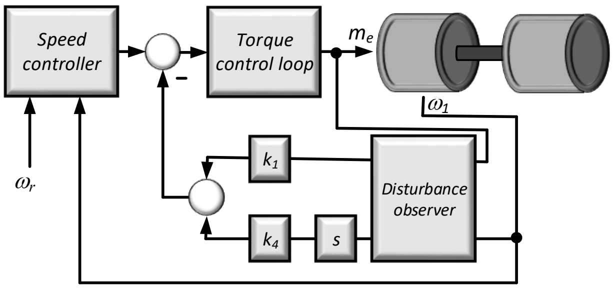

Fig. 2. Block diagram of the control system

The control system coefficients (with the assumption that the optimized transfer function of the electromagnetic-torque control loop is equal to 1) are selected using the following relationships [17] :

$$K_P = 4\xi\omega^3 T_1 T_2 T_c \quad (4)$$

$$K_I = \omega^4 T_1 T_2 T_c \quad (5)$$

$$k_1 = \left(2\omega^2 + 4\xi^2\omega^2\right)T_1 T_c - \omega^4 T_1 T_2 T_c^2 - T_1/T_2 - 1 \quad (6)$$

$$k_4 = 4\xi\omega T_1 T_c - 4\xi\omega^3 T_1 T_2 T_c^2 \quad (7)$$

where: ω – the desired frequency of the system poles (closed-loop), ζ – damping coefficient, k_1, k_2, K_I, K_P – coefficients of the control structure.

The above relationships are determined using the poles placement methodology. The PI controller has the following form:

$$G_r = k_p + k_i/s \quad (8)$$

Formulas (4)-(7) allow shifting poles of the system to any desired location in the complex plane. So, the dynamics of the system can be set freely in the linear range of the operation. This control structure requires information about the torsional torque and its derivative. As it is usually impossible to measure these quantities, it is necessary to use suitable estimators.

Disturbance observers

The disturbance observer for a two-mass system is based on equation (1). The main disadvantage of simulators is sensitivity to initial states and disturbances. However, it should be emphasised that in the applications under discussion, there are no integrators or disturbances (in the simulator structure). Thus, the aforementioned drawbacks are not present in the system under consideration. A block diagram of the classical disturbance observer is shown in Fig. 3a. In practice, the system of Fig. 3b, in which direct differentiation is dispensed with, is often used.

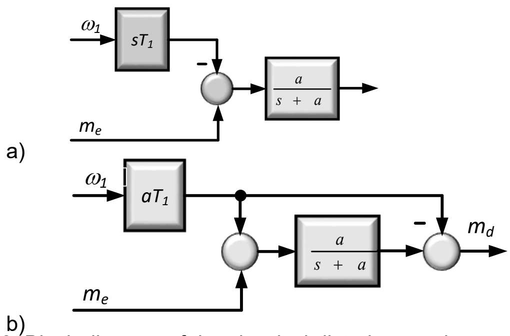

Fig. 3. Block diagram of the classical disturbance observer in direct (a) and transformed (b) form

The properties of the observers depend on the measurement noise levels in the real system, especially in the speed signal. This is due to the presence of a differentiating term in its structure. High levels of noise adversely affect the shape of the disturbance torque estimate. To minimise them, a first-order low-pass filter is introduced into the system. The filter time constant *1/a* depends on the noise level in the particular case. It reduces the noise content, but introduces a delay into the estimated signal.

The disturbance torque can also be determined using the theory of observers. The driving torque m_e is an input signal, and the motor speed ω_1 is an output quantity. The block diagram of the integral disturbance observer is shown in Fig. 4.

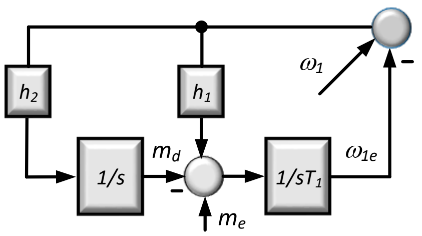

Fig. 4 Block diagram of the reduced observer for disturbance torque estimation

Where h_1, h_2 – are coefficients calculated in (10) and (11); m_d – disturbance torque (here load); m_e – electromagnetic torque; ω_1, ω_{1e} – motor speed: measured and estimated.

The poles placement methodology is applied to determine the observer coefficients. In order to distinguish between control structure and observers, the following description is introduced. The resonant pulsation of the poles is *p*, and the damping coefficient is *a*. The desired polynomial and resulting correction gains have the following forms:

$$p(s) = s^2 + 2aps + p^2 \tag{9}$$

$$h_1 = T_1 2ap \tag{10}$$

$$h_2 = -T_1 p^2 \tag{11}$$

The system is designed with the help of the poles placement methodology. The desired polynomial has the following equation:

$$p(s) = \left(s^2 + 2aps + p^2\right)\left(s + p\right) \tag{12}$$

By comparing the chosen forms of the desired polynomials to the system's characteristic equations, a set of observer coefficients is obtained:

$$h_1 = T_1\left(2ap + p\right), \tag{13}$$

$$h_2 = -T_1\left(2ap^2 + p^2\right), \tag{14}$$

$$h_3 = -T_1 p^3 \tag{15}$$

The presented relationships allow the placement of the system poles at arbitrary positions on the complex plane. Their precise location is a trade-off between observer speed and measurement noise amplification.

In order to reproduce the torque derivative, it is necessary to extend the observer structure with an additional segment, as shown in Fig. 5.

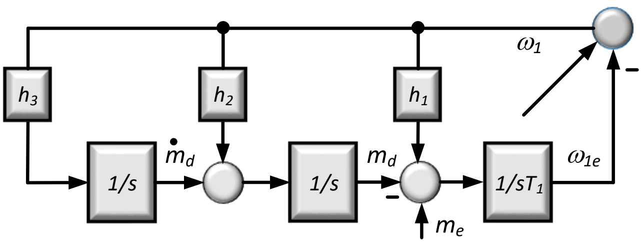

Fig. 5 Block diagram of the reduced observer for disturbance torque and its first derivative estimation

Simulation study

In this chapter, we present simulation outcomes. Fig. 6. a-c shows speed transients for three analysed estimation systems. The first system (a) is a low-pass filter solution. Subfigure (b) presents the case of an observer of disturbance torque, and subfigure (c) uses an observer of disturbance torque and its derivative.

The measured motor speed (black Fig.6 a-c) is fed both to the controller and observer. As it is seen, cases a & c look practically identical, the differences get lost in the noise, case b contains slightly bigger return oscillations after load application.

However, moving towards subfigures d-f, the torsional torque estimated by the considered system and the signal of torsional torque taken from the model are presented. Here, we see that the integral observers are much less noisy and offer much lower delay. This delay can be further improved by tuning the observer towards a specific setup. The subfigures g and h offer a comparison between electromagnetic and torsional torques (from the model).

For further comparison of the analysed systems, the following function was used to assess the quality of the estimation:

$$J = \sum_{i=1}^{n}\left(m_{si} - m_{sei}\right)^2 \tag{16}$$

where: J – quality index, m_{si} – sample of real value of the shaft torque, m_{sei} – sample of estimated value of the shaft torque.

The parameters of the tested observers are determined as follows. The entire possible solution space is searched, and in this way, the coefficients providing the smallest value of the objective function are determined. A damping factor of integral observer poles is assumed to be a=1. Only the value of the resonant pulsation p is changed. The error values are included in Table 1. The input waveforms are presented in Figure 6. a-c for the case of ω=30. The obtained integrated squared error values are included in Table 1. For comparison, percentages are included in the second row. The reference quantity is the classical estimator (Fig. 3).

Table 1. Absolute and relative error values

	System 1	System 2	System 3
ISE(m_s-m_d) ω=30	48.3214	17.6364	9.4143
δ [%]	100%	36.5%	19.48%

The data in Table 1 allow the following conclusions. The system with a classical interference observer (system 1) has the highest error value and provides a reference point for subsequent systems. The simplest integral observer decreases error values by up to 74% (system 2) and lowers noise. Extending its structure to include an integrator that estimates the first derivative substantially reduces estimation errors (system 3) and noise.

Fig. 6 d-f shows fragments from the actual torsional torque waveforms. Also shown are the estimated variables by each system. Analysis of the posted waveforms confirms the conclusions of the error study. The system with the classical solution provides the worst dynamic properties. The estimated waveforms show a high noise level and a delay to the real waveform. The use of a first-order integral observer reduces the visible oscillations. However, the lag in the estimate is still apparent. Using a higher-order observer results in both a low noise level and a significant reduction in lag. In some moments, the estimate is even ahead of the actual waveform. It is important to mention, even worst of this solution provides great performance, the slight differences are seen in the numeric comparison (Table 1.) and during torque estimation (Fig. 6. d-f). But what is important this small gain is obtained at almost no numerical cost.

The subfigure system 1 was presented in subfigures a & d, also as colour red in subfigures g &h. Similarly, system 2 is b, e and green and system 3 is c, f and light blue.

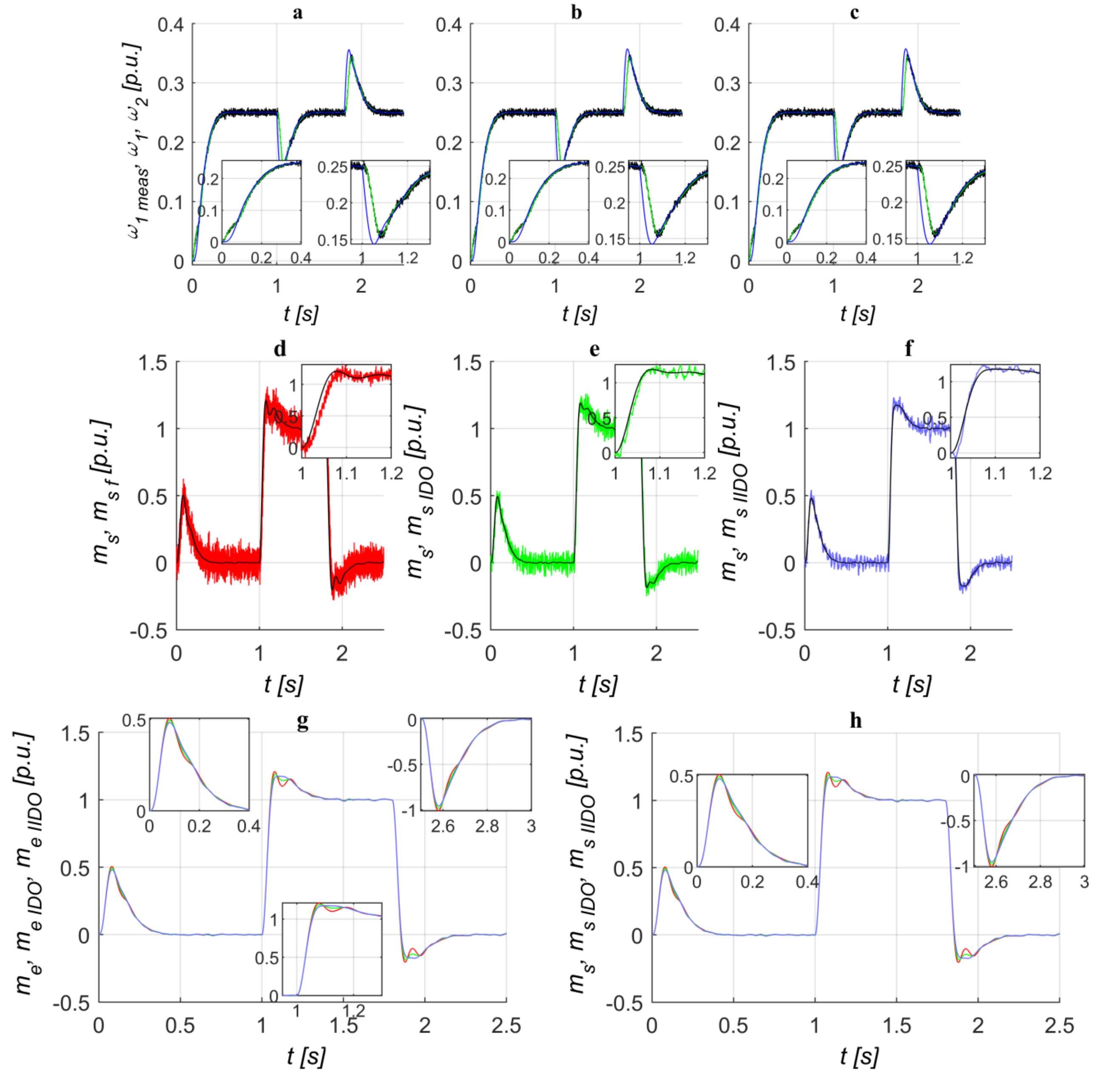

Fig. 6. Transients of measured motor speed ($\omega_{1\ meas}$ black), actual motor speed (green) and load speed (blue) for the control system with 1st observer (a), 2nd observer (b) and 3rd observer (c). Real (black) and estimated (colour) torsional torque for the 1st (d), 2nd (e) and 3rd (f) systems. Electromagnetic torque (g) and torsional torque (h) for the case of 1st system (black), 2nd system (blue) and 3rd system (red) for the case of ζ=1.0, ω=30 and p=90. Colours in subfigures d-f and g-h correspond. Red – 1st system, green 2nd, and blue – 3rd.

Experimental study

To confirm theoretical considerations and simulation studies, bench tests are performed. The test stand (Fig.11) consists of two DC motors connected by a long shaft. The drive motor is fed from a converter with an H-bridge configuration controlled with PWM signal. Both motors are excited using a rectifier. The load is applied by modulating a transistor to a braking resistor. The control algorithm is implemented on a dSpace 1103 control board. The system includes two encoders mounted on two machines. The first (motor) signal is used in the control. The second (load)is used to evaluate the correctness of the working machine. The driving torque is proportional to the measured armature current signal.

A number of experimental tests were performed. Figs. 7-10 show the recorded waveforms of motor and working machine speeds and driving and torsional torques. They confirm the conclusions of theoretical considerations and simulation studies.

On the right side of each figure 7-10 the longer observation period have been shown, and on the left the zoom of chosen time period is presented. The experimental transients follow simulation results. Except for torsional torque, which is unable to be measured using our equipment, instead we show ω_1-ω_2, which is a direct result of torsional torque.

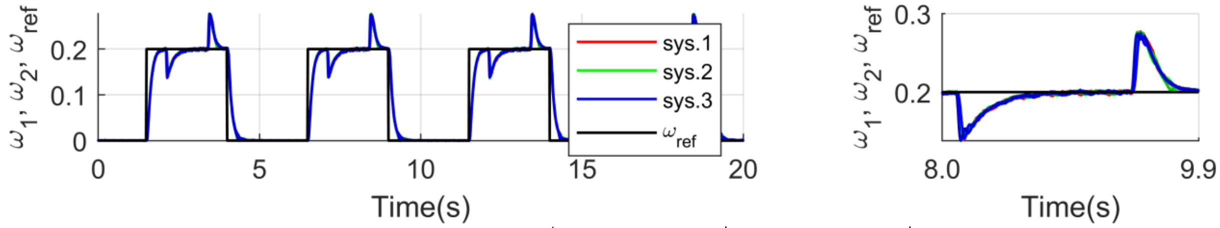

Fig. 7. Experimental transient of drive and load speed for 1st system (blue), 2nd system (green), 3rd system (red), and reference speed. For the case of ζ=1.0, ω=30.

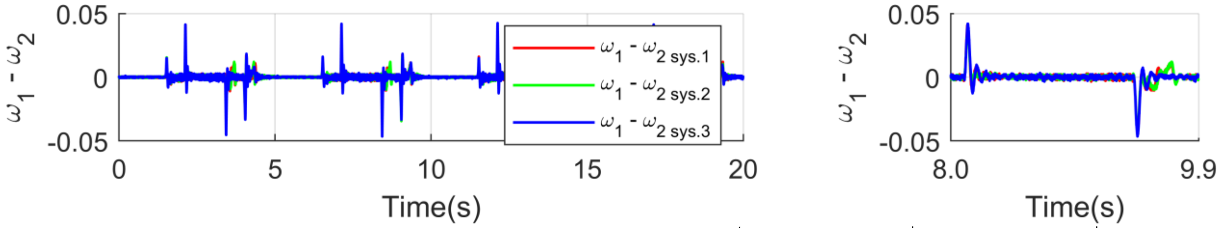

Fig. 8. Experimental transient of difference between drive and load speed for 1st system (blue), 2nd system (green), 3rd system (red), and reference speed.

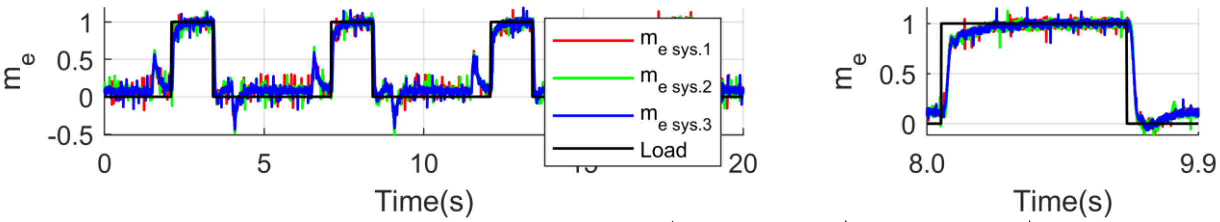

Fig. 9. Experimental transient of measured electromagnetic torque for 1st system (blue), 2nd system (green), 3rd system (red), and reference speed.

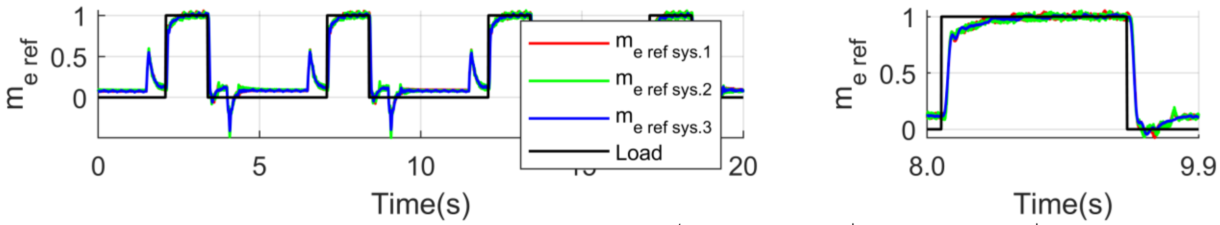

Fig. 10. Experimental transient of reference electromagnetic torque for 1st system (blue), 2nd system (green), 3rd system (red), and reference speed.

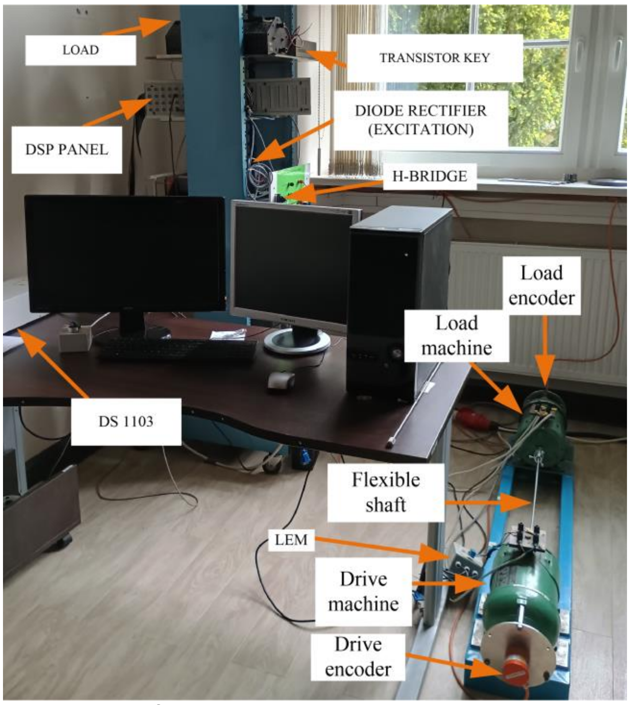

Fig. 11. Photo of experimental setup

Conclusions

The article presents issues related to estimating the disturbing torque in the control structure of the dual-mass system. The main objective of the research is to design and evaluate integral observers. Based on the work carried out, the following summary conclusions can be specified.

- Disturbance observers are commonly used in mechatronic systems. Most often, they include direct differentiation of the velocity signal. This is the classical solution. These observers are characterised by a high level of measurement noise depending on the quality of the available speed sensor. Low-pass filters are used to reduce this, which causes a delay in the estimated torque signal.
- The integral observer of disturbance torque provides a significant reduction of noise in the estimated signal. However, there is a visible delay in the system (as seen in system 1, especially simulation – fig. 6d). As in the classical solution, it is a compromise between the dynamics of the system and the amplification of measurement noise.
- The use of the higher-order integral observers described in the paper improves the quality of state variable estimation. The delay in estimation is almost completely eliminated (as seen in system 3, second-order integral observer, fig. 6f). This system provides the smallest value of the quality index. It decreases almost three times compared to the classical solution.

The disturbance observer needs only the system parameter associated with the drive motor in its structure.

This means that it is immune to changes in parameters related to the shaft and the working machine. Some other observers are also based on load parameters, which makes them more dependent on overall system parameter knowledge.

***Authors**: dr. inż. Piotr Derugo, mgr inż. Amanuel Haftu Kahsay, prof. Krzysztof Szabat, Politechnika Wrocławska, Wydział Elektryczny, piotr.derugo-, amanuelhaftu.kahsay - krzysztof.szabat-@pwr.edu.pl, dr inż. Rafał Zawiślak, Instytut Automatyki, Politechnika Łódzka, mgr Kosuke Shikata, prof. Seiichiro Katsura, Department of the System Design Engineering, Keio University shikata@katsura.sd.keio.ac.jp katsura@sd.keio.ac.jp*

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